Correlation between $pp \to \Delta^+ p$ and $pp \to pp\pi^0$ amplitudes

Venkataraya[1,2,3] and G. Ramachandran[2,4]
[1]Vijaya College, Jayanagar, Bengaluru-560011
[2]G.V.K. Academy, Bengaluru – 560070
[3]Amrita Vishwa Vidyapeetam, Coimbotore-641112
[4]Amrita Vishwa Vidyapeetam,Kasavanahalli, Bengaluru-560035

Introduction

The experimental and theoretical study of $pp \to pp\pi^0$ has excited considerable attention, ever since the total cross section measurements [1] were found to be more than five times larger than the then existing theoretical predictions. Advances in technologies led to complete identification of the three body final state and to measurements employing polarized beam on a polarized target [2]. Many of the early theoretical attempts tried to bridge the gap by introducing several mechanisms taking into account only the lowest partial waves. The Julich meson exchange model which took into consideration higher partial waves as well was however found to be inadequate to provide an overall satisfactory reproduction of the data on $\vec{p}\vec{p} \to pp\pi^0$.

The model independent approach [3] was used to analyze this data [4] and compare the predictions of the Julich model, which revealed among other things [5] the importance of Δ production. The phase ambiguity inherent in [5] was brought out in [6] and a model independent approach to discuss Δ contribution was outlined in [7]. It may also be mentioned that, although the pionic d-wave effects were noticed [8] and the evidence for Ds final state was reported [9], Sd and Ds contributions have been ignored so far in analyzing the data.

The purpose of the present contribution is to take into account Sd and Ds channels in $pp \to pp\pi^0$ improve the discussion in [7] and correlate the resulting $pp \to \Delta^+ p$ and $pp \to pp\pi^0$ amplitudes, so that the importance of Δ contributions can be studied in more detail using the model independent approach.

Theory

Let $(\mathbf{p}_1, \mathrm{E}_1), (\mathbf{p}_2, \mathrm{E}_2)$ and $(\mathbf{q}, \omega)$ denote the four momenta of the two protons and π^0 in the cm frame and $E = E_1 + E_2 + \omega$, the cm energy for the reaction. We use natural units and define an invariant W_1 through

$$W_1^2 = (E_1 + \omega)^2 - |\mathbf{p}_1 + \mathbf{q}|^2 = E^2 + M^2 - 2EE_2 \quad (1)$$

We use Jacobi coordinates

$$\mathbf{R} = \frac{\mathrm{M}\mathbf{r}_1 + \mathrm{m}\mathbf{r}}{\mathrm{M} + \mathrm{m}}; \mathbf{r}' = \mathbf{r} - \mathbf{r}_1; \boldsymbol{\rho} = \mathbf{r}_2 - \mathbf{R} \quad (2)$$

where m and M denoting the pion and nucleon masses. We may express the unpolarized double differential cross section for $pp \to pp\pi^0$ as

$$\frac{d^2\sigma_0}{dW_1 d\Omega_2 d\Omega} \quad (3)$$

where $d\Omega_2$ is the solid angle associated with $\mathbf{p}_2$ and $d\Omega$ is the solid angle associated with $\mathbf{q}$.

We may now focus attention on events with $W_1 = M_\Delta$, where M_Δ denotes the mass of Δ^+.

The reaction matrix [7] for $pp \to \Delta^+ p$ is

$$\mathrm{M} = \sum_{s_i=0}^{1} \sum_{s=1}^{2} \sum_{\lambda=s-s_i}^{s+s_i} \left(S^\lambda(s, s_i) \cdot M^\lambda(s, s_i) \right) \quad (4)$$

where the irreducible tensor amplitudes

$$M_\mu^\lambda(s, s_i) = \sum_\alpha \mathrm{G}_\alpha \mathrm{F}_\alpha \left(\mathrm{Y}_{l_2}(\hat{\mathbf{p}}_2) \otimes \mathrm{Y}_{l_i}(\hat{\mathbf{p}}_i) \right)_\mu^\lambda \quad (5)$$

of rank λ are expressible in terms of partial wave amplitudes

$$\mathrm{F}_\alpha = \mathrm{M}^{j}_{l_2 s; l_i s_i}(\mathrm{E}) \quad (6)$$

at any cm energy E, with α denoting collectively $\alpha = \{l_i, s_i, l_2, s, j\}$.

Conservation of isospin I implies that I can take only one value I=1 and Pauli principle demands that (l_i+s_i) must be even and parity conservation demands that $(-1)^{l_i} = (-1)^{l_f}$. With l_2=0,1, 2, we have a set of thirteen partial wave amplitudes instead of nine in [7], which are listed in Table-1.

TABLE-1: $pp \rightarrow \Delta^+ p$ amplitudes

α	l_2	s	j	l_i	s_i
F_1	0	2	2	2	0
F_2	1	1	0	1	1
F_3	1	1	1	1	1
F_4	1	1	2	1	1
F_5	1	1	2	3	1
F_6	1	2	1	1	1
F_7	1	2	2	1	1
F_8	1	2	2	3	1
F_9	1	2	2	3	1
F_{10}	2	2	0	0	0
F_{11}	2	2	2	2	0
F_{12}	2	2	4	4	0
F_{13}	2	1	2	2	0

The twelve partial wave amplitudes f_i for $pp \rightarrow pp\pi^0$ listed in [6] are now increased to 16 including Sd and Ds channels. The additional four amplitudes are listed in Table-2

TABLE-2: Additional $pp \rightarrow pp\pi^0$ amplitudes

	l	l_f	s_f	j_f	j	l_i	s_i	type
f_{13}	0	2	0	2	2	1	1	Ds
f_{14}	0	2	0	2	2	3	1	Ds
f_{15}	2	0	0	0	2	1	1	Sd
f_{16}	2	0	0	0	2	3	1	Sd

We note that the angular distribution is given in [6] by

$$A^{\lambda}_{\mu} = \left(\left(Y_{l_f}\left(\hat{\mathbf{p}}_f\right) \otimes Y_l\left(\hat{\mathbf{q}}\right)\right)^{L_f} \otimes Y_{l_i}\left(\hat{\mathbf{p}}_i\right)\right)^{\lambda}_{\mu} \quad (7)$$

To correlate the two sets of amplitudes, we express the spherical harmonics $Y_{l_2 m_2}(\hat{\mathbf{p}}_2)$ in terms of solid harmonics

$$\mathcal{Y}_{l_2 m_2}(\mathbf{p}_2) = |\mathbf{p}_2|^{l_2} \cdot Y_{l_2 m_2}(\hat{\mathbf{p}}_2) \quad (8)$$

Likewise, we may express $Y_{l_f m_f}(\hat{\mathbf{p}}_f)$ in (7) in terms of solid harmonics, where $\mathbf{p}_f$ is given by $\mathbf{p}_1 - \mathbf{p}_2 = 2\mathbf{p}_f$. Noting that $\mathbf{p}_1 + \mathbf{p}_2 = -\mathbf{q}$, we may write $-\mathbf{p}_2 = \mathbf{p_f} + \frac{\mathbf{q}}{2}$ and use a well-known formula [10] to write the above in terms of $\left(\mathcal{Y}_{l_f}\left(\mathbf{p}_f\right) \otimes \mathcal{Y}_l\left(\mathbf{q}\right)\right)^{l_2}_{m_2}$. A correlation is readily established between the two sets of amplitudes. Note that $l_2 = l_f + l$ here [10], which corresponds to L_f in [6]. Full details and their implications to assess the Δ contributions to $pp \rightarrow pp\pi^0$ will be presented.

References:

[1] H. O. Meyer *et al.*, Phys. Rev. Lett. **65**, 2846 (1990).

[2] Phys. Rev. Lett. **81**, 3096 (1998); **83**, 5439 (1999); Phys. Rev. C **63**, 064002 (2001).

[3] G. Ramachandran, P.N.Deepak and M.S.Vidya, Phys. Rev. C **62**, 011001(R) (2000); G. Ramachandran, and P.N.Deepak Phys. Rev. C **63**, 051001(R) (2001); P.N.Deepak and G. Ramachandran, Phys. Rev. C **63**, 027601 (2002).

[4] P.N.Deepak, G. Ramachandran and C. Hanhart, Matter and Mater, **21**, 138 (2004); P.N.Deepak, C. Hanhart, G. Ramachandran and M.S. Vidya, Int. J. Mod. Phys. A**20**, 599 (2005).

[5] P.N.Deepak, J.Haidenbauer and C.Hanhart, Phys. Rev. C72, 024004 (2005)

[6] G. Ramachandran, G. Padmanabha and Sujith Thomas Phys. Rev. C **81**, 067601 (2010).

[7] G. Ramachandran, Venkataraya and Sujith Thomas, Proceedings of DAE Symposium on Nuclear Physics 56, (2011).

[8] J.A. Niskanen, Phys. Lett. B**289**, 277 (1992); Y. Maeda et.al., πN Newsl. **13**, 326 (1997).

[9] R. Bilger et.al., Nucl. Phys. A693, 633 (2001).

[10] M.E. Rose, J. Math Phys. 37, 25 (1958).