A field theoretical insight of weak mesonic decay of Λ from hypernuclei

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Theoretical and experimental investigation of Λ decay from hypernuclei are struggling to merge during a long time but they did not succeed yet [1, 2]. Inside the nuclei the mesonic decays will be almost forbidden due to Pauli blocked probability although a small probability may be persisted in the local density approximation [1, 2]. In the present work, we have gone through a field theoretical analysis of $\Lambda \to N\pi$ decay, coming from hypernuclei.

To calculate vacuum width of weak decay $\Lambda \to N\pi$, let us start with the effective weak Lagrangian density,

$$\mathcal{L}^W_{\Lambda N\pi} = iG_F m_\pi^2 \bar{\psi}_N \left(A_\pi + B_\pi\gamma_5\right) \vec{\pi}\cdot\vec{\tau}\psi_\Lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (1)$$

where $G_F m_\pi^2 = 2.21 \times 10^{-7}$ is the weak coupling constant; $A_\pi = 1.05$ and $B_\pi = -7.15$ are empirical constants; ψ_N ψ_Λ and $\vec{\pi}$ are respectively nucleon, Lambda baryon and pion field; $\vec{\tau}$ is the Pauli operator. The isospin spurion $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ [2] is included in order to enforce the empirical $\Delta I = \frac{1}{2}$ rule.

Owing to the Optical theorem, one can obtain this vacuum decay width of $\Lambda \to N\pi$ process from the imaginary part of the vacuum self-energy of Λ for $N\pi$ loop :

$$\Gamma_V(q) = \frac{-1}{16\pi\vec{q}} \int_{\omega_k^-}^{\omega_k^+} d\omega_k L(q, \omega_k, \vec{k}), \quad (2)$$

where

$$\begin{aligned} L(q,k) &= \frac{(G_F m_\pi^2 I_N)^2}{2q} \mathrm{Tr}[(\not{q} + m_\Lambda)(A_\pi - B_\pi\gamma_5) \\ &\quad (\not{q} - \not{k} + m_N)(A_\pi + B_\pi\gamma_5)] \\ &= \frac{2(G_F m_\pi^2 I_N)^2}{q}[A_\pi^2(m_N m_\Lambda + q^2 - q\cdot k) \\ &\quad B\pi^2(-m_N m_\Lambda + q^2 - q\cdot k)] \quad (3) \end{aligned}$$

and $\omega_k^\pm = \frac{R^2}{2q^2}(q_0 \pm \vec{q}W)$, $R^2 = q^2 - m_N^2 + m_\pi^2$, $W = \sqrt{1 - \frac{4q^2 m_\pi^2}{R^4}}$. At the center of mass frame ($q_0 = m_\Lambda$, $\vec{q} = \vec{0}$), we can get experimentally observed [3] ratio of two decay width in vacuum i.e.

$$\begin{aligned} \frac{\Gamma_V(\Lambda \to p\pi^-)}{\Gamma_V(\Lambda \to n\pi^0)} &= \left(\frac{\sqrt{2}}{1}\right)^2 = 2 \\ &= \frac{1.56 \times 10^{-6}\ \mathrm{eV}}{0.78 \times 10^{-6}\ \mathrm{eV}} \quad (4) \end{aligned}$$

because the spurion enforces the isospin factors $I_N = \sqrt{2}$, 1 for $\Lambda \to p\pi^-$, $\Lambda \to n\pi^0$ channels respectively. So the total decay width of $\Lambda \to N\pi$ channels is

$$\Gamma_V = (\sqrt{2})^2\Gamma_V(\Lambda \to p\pi^-) + (1)^2\Gamma_V(\Lambda \to n\pi^0) \quad (5)$$

Now, at finite density decay width of Eq. 2 becomes

$$\begin{aligned} \Gamma_\rho(\vec{q}, \mu_N) &= \frac{1}{16\pi\vec{q}} \int_{\omega_k^-}^{\omega_k^+} d\omega_k \{1 - \theta(\omega_k - \omega_k^{\mathrm{th}})\} \\ &\quad L(q_0 = \omega_q, \vec{q}, k_0 = \omega_k, \vec{k}), \quad (6) \end{aligned}$$

where $\omega_k^{\mathrm{th}}(q_0) = \omega_q - \mu_N$ and $\mu_N = \sqrt{(3\pi^2\rho/2)^{2/3} + m_N^2}$ at density ρ. Due to step

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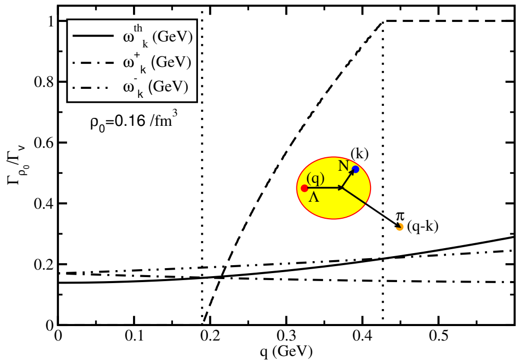

FIG. 1: $\vec{q}$ dependence of Γ_{ρ_0}/Γ_V (dash line) ω_k^+ (dash-double-dotted line), ω_k^- (dash-dotted line) and $\omega_k^{\rm th}$ (solid line) for $\Lambda \to N\pi$ decay.

function θ, we will get

$$\begin{aligned}\Gamma_\rho &= 0 \text{ when } \omega_k^- > \omega_k^{\rm th} \\ &= \Gamma_V - \frac{1}{16\pi\vec{q}}\int_{\omega_k^-}^{\omega_k^{\rm th}} d\omega_k L(q_0 = \omega_q, \vec{q}) \\ &\quad \text{when } \omega_k^- < \omega_k^{\rm th} < \omega_k^+ \\ &= \Gamma_V \text{ when } \omega_k^+ < \omega_k^{\rm th} \ . \end{aligned} \quad (7)$$

In Fig. (1), we have presented the ratio, Γ_ρ/Γ_V as a function of momentum $\vec{q}$, where μ_N = 0.975 GeV is taken for the nuclear matter with its saturation density $\rho = \rho_0 = 0.16$ /fm^3. From the $\vec{q}$ dependence of ω_k^+, ω_k^- and $\omega_k^{\rm th}$, One can identify the origin of three regions of Eq. (7).

Now the bound state of Λ inside the nucleus can be assumed as a complicated quantum mechanical many-body system, which may provide an average momentum $\langle\vec{q}\rangle$ to Λ, when it is going to decay into $N\pi$ channel. From the experimental values of (mesonic) decay widths for different hypernuclei, we can extract the $\langle\vec{q}\rangle$ of Λ inside those hypernuclei. Table (I) shows the numerical band of the $\langle\vec{q}\rangle$'s for the experimental values decay widths, normalized by vacuum widths, inside $^5_\Lambda He$ and $^{12}_\Lambda C$ hypernuclei. Guided from the simplest relation $\langle\vec{q}\rangle = 1/\langle r\rangle$, the quantum mechanical average radius of Bohr radius $\langle r\rangle$ of Λ inside $^5_\Lambda He$ and $^{12}_\Lambda C$ hypernuclei are approximately 0.5 fm and 1 fm, which are too small. It interprets that Λ has a tendency to reside very close to the center of hyper-nucleus, which may be possible as Pauli repulsion in position will not be acted between ψ_Λ and ψ_N (it will only act among the ψ_N's). Our future plan is to investigate the explicit wave functions of Λ inside the hypernuclei, whose average momenta are very close to the extracted values, given in Table (I).

TABLE I: Experimental values of $(\Gamma_A/\Gamma_V)_{\rm exp}$ (second column) and extracted values of $\langle\vec{q}\rangle$ in GeV (third column) for different hypernuclei $^A_\Lambda X$ (first column), having mass number A.

$^A_\Lambda X$	$(\Gamma_A/\Gamma_V)_{\rm exp}$	$\langle\vec{q}\rangle$ (GeV)
$^5_\Lambda He$	$0.59^{+0.44}_{-0.31}$ (BNL [4])	$0.3^{+0.13}_{-0.06}$
$^{12}_\Lambda C$	0.11 ± 0.27 (BNL [4])	0.2 ± 0.06
	0.36 ± 0.13 (KEK [5])	0.255 ± 0.027

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