Mass spectra of Charmonia using Martin-like potential in a Relativistic Dirac formalism

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Introduction

Charmonium has been a powerful tool for the understanding of the strong interaction. Many new charmonium states, including those which are poorly described within the quark model, have been discovered in recent years [1–7]. There is no clear understanding of the region above the threshold, where there are several new Charmonium-like states that are very difficult to accommodate within the spectrum of expected charmonium states. In this context we examine the status of $\psi(3686)$, $\psi(4040)$, $X(4140)$, $\psi(4160)$, $Y(4260)$, $X(4274)$ [8], $Z(4443)$ and $X(4630)$ charmonia like states by looking into the behaviour of the energy level differences of charmonia states and their experimental status.

Methodology

A meson in general is pictured as a color-singlet assembly of a quark and an antiquark independently confined by an average flavor-independent potential of the form [9, 10]

$$V(r) = \frac{1}{2}(1+\gamma_0)(\lambda r^{0.1} + V_0) \qquad (1)$$

This form is taken as a phenomenological representation for the confining interaction expected to be generated by the nonperturbative multigluon mechanism. The two component solution of Dirac equation can be written as

$$\psi_{nlj}(r) = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} \qquad (2)$$

where the positive and negative energy solutions are written as

$$\psi_A^{(+)}(\vec{r}) = N_{nlj} \begin{pmatrix} \frac{ig(r)}{r} \\ \frac{(\sigma.\hat{r})f(r)}{r} \end{pmatrix} \mathcal{Y}_{ljm}(\hat{r}) \qquad (3)$$

$$\psi_B^{(-)}(\vec{r}) = N_{nlj} \begin{pmatrix} \frac{i(\sigma.\hat{r})f(r)}{r} \\ \frac{g(r)}{r} \end{pmatrix} (-1)^{j+m_j-l} \mathcal{Y}_{ljm}(\hat{r}) \qquad (4)$$

and N_{nlj} is the overall normalization constant.

The radial solutions f(r) and g(r) is obtained numerically to yield the energy eigen values. The meson radial wave function for $q\bar{q}$ combination is constructed with the respective quark and anti-quark wave functions given by Eqn. (3) and (4). The parameters are fixed to get the ground state masses of $c\bar{c}$ mesons. The optimised quark mass parameter m_c is taken as 1.27 GeV. Other residual spin dependent interactions are computed perturbatively using the OGEP [11]. The center of mass corrections are incorporated along with the parameter V_0. The parameters λ and V_0 employed in the present study are 1.8078 $GeV^{\nu+1}$ and $-\frac{(1.9098)}{(n+l)^{0.154}}$ GeV respectively.

Results and discussion

We have computed the charmonium and bottonium spectral states which are in good agreement with the reported PDG values of known states. Though there are many excited 1^{--} states of quarkonia known experimentally, most of them beyond 3S states are

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TABLE I: S-wave $c\bar{c}$ spectrum (in MeV)

nS	Present	[3]	[4]	Exp. [1]
1^3S_1	3097.17	3097.14	3097	$\psi(3096.91 \pm 0.011)$
1^1S_0	2973.36	2980.40	2981	$\eta_c(2980.3 \pm 1.2)$
2^3S_1	3682.21	3689.95	3778	$\psi'(3686.09 \pm 0.04)$
2^1S_0	3613.57	3633.49	3666	$\eta_c'(3637 \pm 4)$
3^3S_1	4029.37	4030.32	4069	$\psi(4040)$
3^1S_0	3980.85	3991.99	3991	-
4^3S_1	4277.04	4273.49	4240	$X(4274)$ [8] or $Y(4260)$
4^1S_0	4238.96	4244.11	4180	-
5^3S_1	4469.14	4464.12	4478	$Z(4443^{+24}_{-18})$
5^1S_0	4437.66	4440.12	4424	-
6^3S_1	4626.45	4621.56	4690	-
6^1S_0	4599.22	4601.19	4640	-

TABLE II: P-wave $c\bar{c}$ spectrum (in MeV)

State	Present	[3]	[4]	Exp. [1]
1^3P_2	3554.26	3570.00	3552	$\chi_{c2}(3556.20 \pm 0.09)$
1^3P_1	3517.80	3490.94	3542	$\chi_{c1}(3510.66 \pm 0.07)$
1^3P_0	3391.13	3392.11	3489	$\chi_{c0}(3414.75 \pm 0.31)$
1^1P_1	3502.88	3523.88	3542	$h_c(3525.42 \pm 0.29)$
2^3P_2	3967.80	3949.01	3970	$\chi_{c2}(3927.2 \pm 2.6)$
2^3P_1	3876.99	3902.55	3958	-
2^3P_0	3589.80	3844.49	3912	-
2^1P_1	3894.78	3921.91	3960	-
3^3P_2	4268.86	4211.78	-	-
3^3P_1	4128.17	4178.47	-	-
3^3P_0	3693.37	4136.84	-	-
3^1P_1	4166.54	4192.35	-	-

still not understood completely. And in the case of P-wave states only 1^3P_J, 1^1P_1, and 2^3P_2 of the charmonia are known experimentally.

The states $\psi(4040)$, $Y(4260)$ or $X(4274)$, $Z(4443)$ are close to our predicted 3S, 4S and 5S states, while $\psi(3686)$, $X(4140)$, $\psi(4160)$, $X(4630)$, etc., do not match with the predicted results. Figure 1 shows the energy level diagrams of charmonium pectra along with the reported experimental results.

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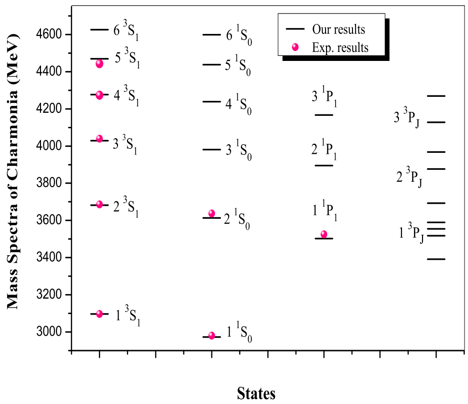

FIG. 1: Charmonium Spectroscopy

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