Equation of state of a strongly coupled Quark Gluon Plasma using Cornell Potential

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Introduction

Experiments carried out at RHIC have shown that Quark Gluon Plasma(QGP) is strongly coupled at temperatures below $3T_c$ [1]. In this work we will investigate whether the Cornell Potential model, which has so far given encouraging results in dealing with QGP[2] [3], can be used to study strongly coupled QGP or sQGP. We will study sQGP using Cornell Potential by considering QGP as consisting of bound states of quarks. We will compare the equation of state thus obtained with lattice data. We proceed by using an approximate solution of Schroedinger equation with Cornell potential and using it to evaluate the second Virial Coefficient quantum mechanically.

Approximate solution for quark-antiquark pair in Cornell Potential

The Cornell Potential between quarks and anti-quarks is given by,

$$V(r) = ar - \frac{b}{r}. \tag{1}$$

This has been a successful phenomenological potential in describing the mass spectra of mesons and is known to be a good model of the QCD interaction. The Schroedinger equation with Cornell Potential does not admit exact solutions. So one has to resort to approximation methods. One such method that is useful is known as the Nikiferov-Uvarov method. We make use of the following solution [4].

$$E_{nl} = \frac{3a}{\delta} - \frac{2\mu\left(b + \frac{3a}{\delta^2}\right)^2}{\left[(2n+1) \pm \sqrt{1 + 4l(l+1) + \frac{8\mu a}{\delta^3}}\right]^2}, \tag{2}$$

where, δ is a parameter equal to the inverse of what is referred to as the characteristic radius r_0. This solution is shown to reproduce the mass spectra of heavy mesons successfully with suitable choices of the parameters a, b, and δ. n takes the values 0, 1, 2, etc. and is related to the principal quantum number $\tilde{n}$ by the relation $\tilde{n} = n + l + 1$.

Quantum Theory of the second Virial Coefficient

Pais and Uhlenbeck have obtained an exact quantum mechanical expression for the second Virial Coefficient[5]. We use it in the form of given in[6].

$$b_2 - b_2^0 = 8^{1/2} \sum_B e^{-\beta\epsilon_B} + \frac{8^{1/2}}{\pi} \sum_l{}' (2l+1) \int_0^\infty dk \Big[e^{-\beta\hbar^2 k^2/m} \times \frac{\partial \eta_l(K)}{\partial k} \Big],$$

where, b_2, b_2^0 are the second cluster integrals for the interacting and non-interacting cases respectively. ϵ_B are the bound states and η_l is the phase shift during scattering. Since we deal only with the bound states, we can take the phase shifts to be equal to zero[5] . Thus for our case,

$$b_2 - b_2^0 = 8^{1/2} \sum_B e^{-\beta\epsilon_B}, \tag{3}$$

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where, the summation goes over all bound states made possible by the two body interaction. The second Virial coefficient is related to the second cluster integral by,

$$b_2 = -a_2. \quad (4)$$

The equation of state for sQGP

Once the second cluster integral is obtained, we can construct the equation of state of the system using,

$$\frac{P}{KT} = \frac{1}{\lambda^3} \sum_{l=1}^{\infty} b_l z^l, \quad (5)$$

where, z is the fugacity which is taken equal to 1. We shall take up to the second term in the above infinite sum. Thus,

$$\frac{P}{KT} = \frac{1}{\lambda^3} [b_1 + b_2]. \quad (6)$$

$b_1 = 1$, and b_2 can be evaluated by using equation (2) in equation(3) and noting that b_2^0 is equal to $-\frac{1}{2^{5/2}}$ for Fermions[6]. Using b_2 in equation (6), we have plotted P/T^4 against T taken in GeV (see Figure 1). We have chosen $\sqrt{a} = .449GeV$, $b = .1$, $\delta = .840GeV$. The summation over the bound states is carried out upto $n = 885$. The mass $M = .0016GeV$ is chosen as the reduced mass corresponding to the masses of up and down quarks. In Figure 1, we have also compared our equation of state with the lattice data as given in [7].

Results and discussions

In this work we have obtained an equation of state of QGP, assuming it to be composed of bound states of quarks. This treatment gives a curve for the equation of state which is compared with the lattice result corresponding to two flavour QGP and the agreement seems to be good up to around $3T_c$ with $T_c = .160GeV$. Thus we conclude that treating QGP as a collection of bound states can give the correct form of equation of state even above the transition temperature T_c. Our EOS shows deviation from the lattice data around $3T_c$, suggesting that the interaction becomes weak around this temperature. This result is also in agreement with the criterion for sQGP as given by Shuryak [1]. We have shown that the Cornell Potential model can be successfully applied to studying sQGP.

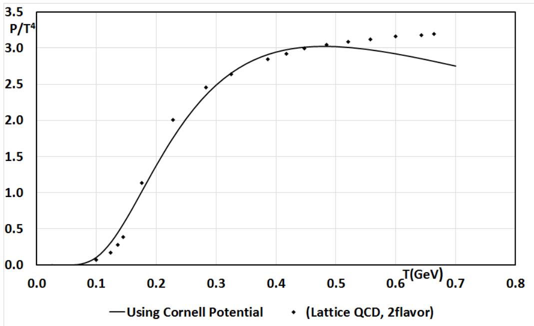

FIG. 1: Plots of P/T^4 as a function of T(in GeV) using the Cornell potential, and lattice result for 2 flavour QGP

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References

[1] E. Shuryak, arXiv:hep-ph/0312227v2 (2003).
[2] K. M. Udayanandan, P. Sethumadhavan, and V. M. Bannur, Physical Review C (2007).
[3] B. Sheikholeslami-Sabzevari, Phys.Rev.C **65**, 054904 (2002).
[4] S. M. Kuchin and N. V. Maksimenko, Universal Journal of Physics and Application 1 **3**, 295 (2013).
[5] A. Pais and G. E. Uhlenbeck, Physical Review **116** (1959).
[6] R. K. Pathria and P. D. Beale, *Statistical Mechanics* (Butterworth-Heinemann, 2011).
[7] E. Laermann and O. Philipsen, arXiv:hep-ph/0303042v1 (2003).