# Validity of Parabolic approximation in the study of crust-core transition density using Skyrme interaction

K. Madhuri[1, 2,*], S. P. Pattnaik[1, 3], T.R. Routray[1], D. N. Basu[4] and B. Behera[1]
[1]School of Physics, Sambalpur University, Jyotivihar-768 019, INDIA
[2] Govt. Women's Jr. College, Sambalpur-768001, Odisha, INDIA
[3] Department of Physics, Govt. Junior College, Sundargarh , Odisha, 770001, INDIA
[4] Variable Energy Cyclotron Center, 1/AF Bidhan Nagar, Kolkata, 700064, INDIA
* email:kmadhuriphy@gmail.com

## Introduction

The equation of state (EoS) of isospin asymmetric nuclear matter (ANM) plays a central role in understanding the structure of radioactive nuclei, the reaction dynamics induced by rare isotopes, the liquid-gas phase transition in asymmetric nuclear matter and many critical issues in astrophysics. The isospin dependent part of the energy per particle in isospin asymmetric nuclear matter (ANM) is considered as the nuclear symmetry energy $E_s(\rho)$ and the density ρ dependence of it is poorly known. One of the popular ways of expressing the energy density of ANM is the Taylor series expansion of the energy density in the even powers of the asymmetry parameter β. As per the result obtained by various model calculations, the energy density of the ANM is expressed in a parabolic expression as the fourth and higher-order terms in the Taylor's series expansion of energy have negligible contribution to total energy density. However the importance of higher order terms cannot be ignored for determining some physical quantities under special conditions. One of such situation is the prediction of the transition density signifying the onset of the phase transition between the uniform dense liquid core and the solid crust in neutron stars. This is an interesting study made by limited number of workers in the current decade [1-4]. In this work, the critical importance of the 4$^{th}$ order, 6$^{th}$ order and 8$^{th}$ order terms in the Taylor's series expansion in the prediction of the crust-core transition density has been examined in the Non-relativistic approximation using expressions of the energy density for Skyrme interaction [5]. The thermodynamics method has been used to calculate the crust-core phase transition in neutron star.

## Formalism

The Taylor series expansion of the energy density of ANM, H (ρ, β), is

$$H(\rho,\beta) = H(\rho) + \beta^2 E_{sym,2}(\rho) + \beta^4 E_{sym,4}(\rho) + \beta^6 E_{sym,6}(\rho) + \beta^8 E_{sym,4}(\rho) + \cdots \quad \ldots(1)$$

where, H (ρ) is the energy density of symmetric nuclear matter (SNM) at density $\rho(=\rho_n+\rho_p)$, $\rho_n$ and $\rho_p$ being the neutron and proton densities and $\beta = \frac{\rho_n-\rho_p}{\rho_n+\rho_p}$ is the isospin asymmetry parameter and $E_{sym,m}(\rho) = \frac{1}{m!}\frac{\partial^m H(\rho,\beta)}{\partial\beta^m}|_{\beta=0}$, m=2,4,6,8,…. The core of the neutron star is dominantly composed of β-equilibrated dense n+p+e+μ matter in liquid phase and matter is as a whole charge neutral. The conditions for β-equilibrium on charge neutrality are

$$\mu_n - \mu_p = \mu_e = \mu_\mu \quad \ldots(2)$$

$$Y_p = Y_e + Y_\mu, \quad \ldots(3)$$

where, $\mu_i$ and $Y_i$, i=n, p, e, μ are the respective chemical potentials and particles fractions. Here n, p, e and μ represent neutron, proton, electron and muon respectively.

The transition density is calculated by analyzing the thermodynamical stability conditions in the β- equilibrated dense n+p+e+μ matter which is given by [1,2],

$$V_{th} = \left[2\rho\frac{\partial e}{\partial\rho} + \rho^2\frac{\partial^2 e}{\partial\rho^2} - \rho^2\left(\frac{\partial^2 e}{\partial\rho\partial Y_p}\right)^2\left(\frac{\partial^2 e}{\partial {Y_p}^2}\right)^{-1}\right]$$

... (4)

where, $e = \frac{H(\rho,\beta)}{\rho}$ is the energy per baryon, $V_{th}$ is $\left(\frac{\partial P}{\partial \rho}\right)_{\mu}$ and P is the baryonic pressure. The expressions of the energy density for Skyrme interaction [5] is given by

$$H(\rho_n, \rho_p) = \frac{\hbar^2}{2M}(\tau_n + \tau_p) + \frac{t_0}{4}[(2+x_0)\rho^2 - (1+2x_0)(\rho_n^2 + \rho_p^2)] + \frac{t_3}{24}\rho^{\gamma}[(2+x_3)\rho^2 - (1+2x_3)(\rho_n^2 + \rho_p^2)] + \frac{1}{8}[t_2(1+2x_2) - t_1(1+2x_1)](\tau_n\rho_n + \tau_p\rho_p) + \frac{1}{8}[t_1(2+x_1) + t_2(2+x_2)]\tau\rho, \quad \ldots (5)$$

where, $\tau_{n(p)}$ is the neutron (proton) kinetic energy density and $\tau = \tau_n + \tau_p$.

Now the stability conditions for the exact expression, the 2nd, 4th, 6th and 8th order approximation of the energy density in Taylor series expansion in equation (1) have been examined to calculate the transition density.

**Results and Discussion**

Using the β-stability for the exact expression, 2nd, 4th, 6th and 8th order approximation the proton fraction and lepton fractions at different densities for the exact expression, 2nd, 4th, 6th and 8th order approximations are obtained from solutions of equations (2) and (3) simultaneously. Using the value of proton fraction thus obtained, the energy per particle of the baryonic part in the β- stable matter is calculated for the EoS of Skyrme interaction.

The parameters of EoS of Skyrme interaction of Sly4, SKm*, LNS, SLy7, KDE0v1, G_sigma, SkMP, SKX and NRAPR have been taken from Reference [6]. Although the higher order contributions have little influence so far as the energy density is concerned, its important role in the prediction of the crust core transition densities, $\rho_t^{exa}$, $\rho_t^{2nd}$, $\rho_t^{4th}$, $\rho_t^{6th}$ and $\rho_t^{8th}$ calculated from Equation (4) for exact expression ,2nd ,4th ,6th and 8th order approximations, respectively, can be seen from Table-1. The comparison of the results of these two approximation shows that the exact expression, 4th ,6th and 8th order terms lowers the prediction of the transition density as compared to the 2nd order one.

| EOS | $\rho_t^{exa}$ | $\rho_t^{2nd}$ | $\rho_t^{4th}$ | $\rho_t^{6th}$ | $\rho_t^{8th}$ |
|---|---|---|---|---|---|
| Sly4 | .086 | .094 | .093 | .092 | .091 |
| SKm* | .086 | .097 | .095 | .093 | .092 |
| LNS | .086 | .101 | .098 | .096 | .094 |
| SLy7 | .087 | .093 | .091 | .090 | .089 |
| KDE0v1 | .090 | .096 | .095 | .094 | .093 |
| G_sigma | .061 | .096 | .089 | .085 | .082 |
| SkMP | .071 | .091 | .087 | .084 | .082 |
| SKX | .101 | .107 | .105 | .104 | .103 |
| NRAPR | .100 | .107 | .104 | .103 | .101 |
| EOS | $P_t^{exa}$ | $P_t^{2nd}$ | $P_t^{4th}$ | $P_t^{6th}$ | $P_t^{8th}$ |
| Sly4 | .389 | .452 | .439 | .428 | .415 |
| SKm* | .463 | .597 | .583 | .560 | .531 |
| LNS | .558 | .785 | .741 | .702 | .665 |
| SLy7 | .382 | .449 | .435 | .423 | .410 |
| KDE0v1 | .500 | .589 | .568 | .551 | .534 |
| G_sigma | .259 | .993 | .815 | .705 | .622 |
| SkMP | .336 | .665 | .598 | .548 | .502 |
| SKX | .527 | .572 | .577 | .566 | .536 |
| NRAPR | .591 | .639 | .644 | .624 | .597 |
| EOS | $Y_P^{exa}$ | $Y_P^{2nd}$ | $Y_P^{4th}$ | $Y_P^{6th}$ | $Y_P^{8th}$ |
| Sly4 | .036 | .0347 | .0362 | .0366 | .0368 |
| SKm* | .036 | .0344 | .0358 | .0363 | .0365 |
| LNS | .0298 | .0298 | .0315 | .0316 | .0314 |
| SLy7 | .0363 | .0344 | .0358 | .0363 | .0365 |
| KDE0v1 | .0415 | .0399 | .0415 | .0418 | .0419 |
| G_sigma | .0139 | .0187 | .0186 | .0179 | .0171 |
| SkMP | .0185 | .0202 | .0216 | .0215 | .0213 |
| SKX | .0414 | .0375 | .0406 | .0413 | .0414 |
| NRAPR | .0423 | .0437 | .0429 | .0423 | .0419 |

**Table I** Results of transition density, Pressure and proton fraction for the exact expression, 2nd, 4th, 6th and 8th order .

**References**


[1] S. Kubis,2007, Phys. Rev. C 76, 025801

[2] J. Xu, L. W. Chen, B.A. Li and H.R.Ma, 2009, Astrophys. J. 697, 1549

[3] B.J. Cai and L.W. Chen, 2012, Phys.Rev.C 85, 024302

[4] D. Atta and D.N. Basu, 2014, Phys.Rev.C 90, 035802

[5] Dutra M *et al* 2012 *Phys. Rev.* C **85** 035201

[6] J. Friedrich and P.-G. Reinhard,1986, Phys.Rev.C,33,1.