Anharmonic sextic potential and shape phase transition in nuclei

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Introduction

The highly correlated motion of nucleons in nuclei on which the collective nucleon model is based can explain various structural and reaction data in an efficient way [1]. Within the framework of collective nuclear model, the atomic nuclei acquire well defined shape which may change due to several dynamical reasons. For instance,in an isotopic chain it is possible that some of the isotopes have spherical shape while others have well deformed shape indicating that there exists some critical point which separate the region of different shapes. This shape phase transition have attracted significant attention and play an important role in understanding the collective model. One of the effective way to study the shape phase transition is to employ Bohr Hamiltonian in conjunction with sextic potential which consists in solving a Schrödinger like equation in deformation parameter space [2, 3]. In general this equation is not solvable however, in the present work, we have solved this equation using power series method as discussed below.

Formalism

Consider the three-dimensional radial Schrödinger equation with a potential V(r) that can be written as

$$\Big[\frac{d^2}{d\,r^2}+\frac{2}{r}\frac{d}{d\,r}-\frac{l(l+1)}{r^2}+\frac{2\mu}{\hbar^2}\Big(E-V(r)\Big)\Big]\,R(r)=0, \qquad (1)$$

where $l = 0, 1, 2, ...$ is the angular momentum quantum number, μ is reduced mass and E denote the energy eigenvalue of the system.

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Now the anharmonic potential is assumed to have the following form

$$V(r) = a\,r^2 - b\,r^4 + c\,r^6; \qquad (2)$$

for which Eq. (1) becomes

$$\frac{d^2\,R}{d\,r^2}+\frac{2}{r}\frac{dR}{d\,r}-\frac{l(l+1)}{r^2}\,R(r)+(\epsilon-a_1\,r^2+b_1\,r^4-c_1\,r^6)\,R(r)=0, (3)$$

where $\epsilon = \frac{2\mu\,E}{\hbar^2}$, $a_1 = \frac{2\,\mu a}{\hbar^2}$, $b_1 = \frac{2\,\mu b}{\hbar^2}$, $c_1 = \frac{2\,\mu c}{\hbar^2}$.
Consider an ansatz for the radial wave function [4]

$$R(r) = e^{-\alpha\,r^2+\beta\,r^4}\,F(r) \qquad (4)$$

where α and β are positive constants. Using Eq.(4)in Eq.(3) we get

$$F^{''}(r)+\Big[-4\alpha\,r+8\,\beta\,r^3+\frac{2}{r}\Big]F^{'}(r)+\Big[(\epsilon-6\alpha)+(4\alpha^2+20\,\beta-a_1)r^2+(-16\alpha\beta+b_1)r^4+(16\beta^2-c_1)r^6-\frac{l(l+1)}{r^2}\Big]\,F(r)=0. \qquad (5)$$

The functional form of $F(r)$ is written as

$$F(r)=\sum_{n=0}^{\infty}a_n r^{n+l}, \qquad (6)$$

Substituting equation (6) into (5) we obtain

$$\sum_{n=0}^{\infty}a_n\Bigg[\{(n+l)(n+l-1)+2(n+l)-l(l+1)\}r^{n+l-2}+\{-4\alpha(n+l)+\epsilon-6\alpha\}r^{n+l}+\{8\beta(n+l)+4\,\alpha^2+20\,\beta-a_1\}r^{n+l+2}+\{-16\alpha\beta+b_1\}r^{n+l+4}+\{16\beta^2-c_1\}r^{n+l+6}\Bigg]=0(7)$$

Equating each coefficient of r in eq. (7) to zero, we obtain the following relations

$$(n + l)(n + l + 1) - l(l + 1) = 0; \quad (8)$$

$$\epsilon_r = 2\alpha(2n + 2l + 3); \quad (9)$$

$$\alpha = \frac{b_1}{4\sqrt{c_1}}; \quad (10)$$

$$\beta = \frac{\sqrt{c_1}}{4}; \quad (11)$$

$$a_1 = 4\beta(2n + 2l + 5) + 4\alpha^2. \quad (12)$$

After solving Eqs. (8)-(11), the energy eigenvalues and corresponding eigenfunction becomes

$$E_{nl} = \frac{b}{2}\sqrt{\frac{\hbar^2}{2\mu\ c}}(2n + 2l + 3) \quad (13)$$

$$R(r) = e^{-\alpha\ r^2 + \beta\ r^4} \sum_{n=0}^{\infty} a_n r^{n+l}, \quad (14)$$

and the parameter a of potential (2) satisfy the following restriction

$$a = \sqrt{\frac{\hbar^2\ c}{2\mu}}(2n + 2l + 5) + \frac{b^2}{4\ c} \quad (15)$$

The energy spectra of the system in 2-dimensions, after changing $l = m - 1/2$ is written as [5]

$$E_{nl} = b\sqrt{\frac{\hbar^2}{2\mu\ c}}(n + m + 1) \quad (16)$$

with the restriction on potential parameter a satisfy

$$a = \sqrt{\frac{\hbar^2\ c}{2\mu}}(2n + 2m + 4) + \frac{b^2}{4\ c} \quad (17)$$

Conclusion

In the present work, we have analytically solved the radial SE in three dimensions with anharmonic sextic potential using power series technique and have obtained the complete eigenvalue spectra. The results presented here are the preliminary and the method may be extended to study shape phase transition in nuclei.

References

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