Searching for a universal correlation among symmetry energy parameters

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Introduction

Symmetry energy parameters have profound implications in terrestrial nuclear physics as well as in astrophysics and cosmology, as they carry the signatures of isovector part of the nuclear interaction. Symmetry energy coefficient can be defined as,

$$C_2(\rho) = \frac{1}{2}\left(\frac{\partial^2 e(\rho,\delta)}{\partial\delta^2}\right)_{\delta=0}, \tag{1}$$

where $e(\rho,\delta)$ is the energy per nucleon of infinite nuclear matter (NM) with density ρ $(= \rho_n + \rho_p)$ and isospin asymmetry $\delta \left(= \frac{\rho_n-\rho_p}{\rho}\right)$, where, $\rho_n(\rho_p)$ is the neutron (proton) density. The symmetry energy coefficient can be expanded around the saturation density ρ_0 $(= 0.16\ \text{fm}^{-3})$ as,

$$C_2(\rho) \simeq C_2^0 + L_0\epsilon_0 + \frac{1}{2}K^0_{sym}\epsilon_0^2 + \frac{1}{6}Q^0_{sym}\epsilon_0^3. \tag{2}$$

Here, C_2^0 is symmetry energy coefficient at ρ_0, L_0 is slope parameter related to the first derivative of $C_2(\rho)$ with respect to density ρ. Similarly, K^0_{sym} and Q^0_{sym} are connected to the second and third derivative of $C_2(\rho)$ with respect to ρ called the curvature parameter and the skewness parameter, respectively.

C_2^0 is known in quite tight bound (32±3 MeV) from precisely known experimental data on binding energies of nuclei. However, L_0 is not that well-constrained. From different experimental constraints the present accepted range of L_0 is 35 MeV $< L_0 <$ 85 MeV. The uncertainty becomes even larger when one tries to constrain the value of K^0_{sym} or Q^0_{sym}. Till date there is no constraint from experiment is available to determine the values of K^0_{sym} or Q^0_{sym} precisely. Across various relativistic and non-relativistic mean-field models, the values of K^0_{sym} and Q^0_{sym} lie over a very wide range -700 MeV $< K^0_{sym} <$ 400 MeV and -800 MeV $< Q^0_{sym} <$ 1500 MeV [1, 2]. Using a representative set of relativistic and non-relativistic mean-field models an empirical linear correlation between L_0 and K^0_{sym} is suggested [3, 4]. However, the results seem to be dependent on the choice of set of models. We address this problem in this contribution by deriving analytical relations among different symmetry energy elements with in a simplistic model and validate those relations by using 500 relativistic and non-relativistic mean-field models used in the literature.

Theoretical Model

Starting with some basic equations of statistical mechanics and using few reasonable assumptions we obtained an energy density functional (EDF) of NM with density ρ and asymmetry δ [5]. Using this this EDF expressions for K^0_{sym} and Q^0_{sym} can be written as,

$$\begin{aligned} K^0_{sym} &= -3\alpha\left[3C_2^0 - L_0\right] + E_F^0[(3\alpha-4) \\ &\quad + \left(\frac{2}{3}\frac{m}{m_0^*} + k_-\rho_0\right)(5-3\alpha)]; \end{aligned} \tag{3}$$

$$\begin{aligned} Q^0_{sym} &= 15\alpha\left[3C_2^0 - L_0\right] + K^0_{sym}(3\alpha-1) \\ &\quad + E_F^0(2-3\alpha). \end{aligned} \tag{4}$$

Here, α is given by,

$$\alpha = \frac{\frac{K_0}{9} + \frac{E_F^0}{3}\left(\frac{12}{5} - 2\frac{m}{m_0^*}\right)}{E_F^0\left(3 - 2\frac{m}{m_0^*}\right) - e_0}. \tag{5}$$

In Eq.(3-5), E_F^0 is the fermi energy, m_0^* (m) is the effective (free) nucleon mass, e_0 and K_0 are energy per particle and nuclear incompressibility of symmetric NM respectively, all of them being defined at ρ_0. k_- determines the difference between neutron and proton effective masses at ρ_0.

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Results and discussion

Equation (3) throws a hint that K^0_{sym} should be linearly correlated with $(3C^0_2 - L_0)$. In Fig. (1) we plotted K^0_{sym} as a function of L_0 (left panel) and $(3C^0_2 - L_0)$ (right panel) for 500 different mean-field models, both relativistic and non-relativistic, used in the literature. The improvement in the correlation from the left panel to the right panel is realized by looking at the correlation coefficients, as indicated by r in the figure. We have also seen that this correlation between K^0_{sym} and $(3C^0_2 - L_0)$ does not depend on the choice of set of models (not shown here) unlike K^0_{sym} - L_0 correlation, which shows its universal nature.

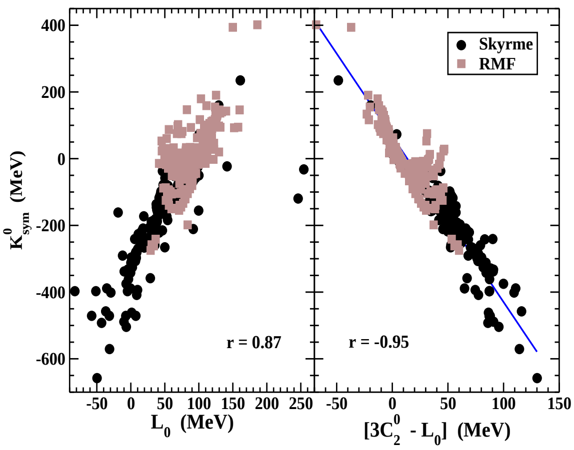

FIG. 1: K^0_{sym} plotted against L_0 (left) and $(3C^0_2 - L_0)$ (right) for 500 mean-field models. The straight line fitted to the universal correlation obtained in the right is given by $K^0_{sym} = d_1(3C^0_2 - L_0) + d_2$ with $d_1 = -4.97 \pm 0.07$ and $d_2 = 66.8 \pm 2.1$.

Inspired by Eq. (4), we plotted Q^0_{sym} as a function of $(3C^0_2 - L_0)$ in the left panel of Fig. (2). However, the correlation is relatively weaker ($r = 0.66$) compared to that of between K^0_{sym} and $(3C^0_2 - L_0)$. The primary reason behind that is the error propagation from K^0_{sym} appearing in the right hand side of Eq. (4). Moreover, different symmetric NM properties (K_0, m^*_0, E^0_F and eventually α) have quite significant contributions in the value of Q^0_{sym}, which have some variation across concerned 500 mean-field models. If we impart a reasonable limit on m^*_0 and K_0 as 0.65 $< \frac{m^*_0}{m} <$ 0.85 and 200 MeV $< K_0 <$ 260 MeV and plot Q^0_{sym} versus $(3C^0_2 - L_0)$ for those models which follow these limits, show a very high linear correlation ($r = 0.93$). It again points out the near-universality in the correlations proposed by the simplistic model described above.

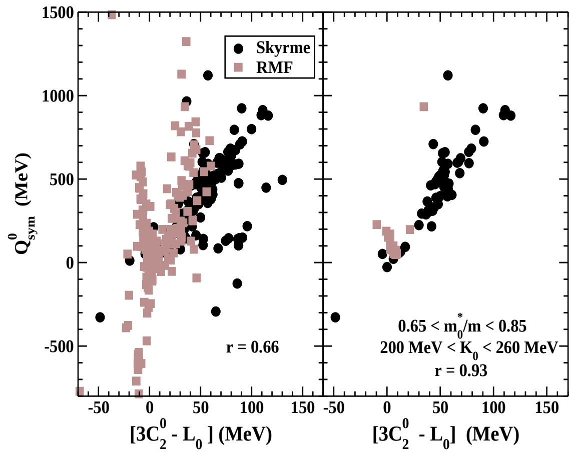

FIG. 2: Q^0_{sym} plotted against $(3C^0_2 - L_0)$ for 500 mean-field models (left) and for ∼ 200 mean-field models, chosen by constraints on m^*_0 and K_0 (right).

Summary and conclusion

By using a simple model based on the basic rules of statistical mechanics, we suggested possible correlations among different symmetry energy parameters. The correlations were realized in practice by using 500 different mean-field models often used in the literature. This shows the near-universality in the correlations we proposed and holds a promise to narrow down the so-far undetermined higher order symmetry energy parameters by using the precise informations on the lower order symmetry parameters.

References

[1] M. Dutra, O. Lourenço et. al., Phys. Rev. C **85**, 035201 (2012).
[2] M. Dutra, O. Lourenço et. al., Phys. Rev. C **90**, 055203 (2014).
[3] I. Vidaña, C. Providência et. al., Phys. Rev. C **80**, 045806 (2009).
[4] J. Dong, W. Zuo et. al., Phys. Rev. C **85**, 034308 (2012).
[5] C. Mondal, B. K. Agrawal et. al., Phys. Rev. C **96**, 021302(R) (2017).