Quasiparticle viscous hydrodynamics from kinetic theory

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Introduction

Relativistic viscous hydrodynamics has been very successful in explaining a wide range of collective phenomena observed in high-energy heavy-ion collisions [1]. In order to derive the hydrodynamic evolution equations, one typically resorts to a simple microscopic theory such as the kinetic theory. On the other hand, in order to incorporate realistic properties of the strongly-interacting matter in the hydrodynamic evolution, equation of state and transport coefficients obtained from lattice QCD are implemented. However, the latter procedure inadvertently fixes the parameters of the microscopic theory, introducing effective interactions which were not taken into account in the derivation of the hydrodynamic evolution equations. These inconsistencies eventually lead to the violation of thermodynamic relations in such a system.

In this proceedings contribution, we construct an effective quasiparticle kinetic theory which can reproduce any thermodynamically consistent equation of state. We then present the derivation of evolution equations for second-order viscous hydrodynamics, and the corresponding transport coefficients, for a system of single species of quasiparticles from an effective Boltzmann equation in the relaxation time approximation.

Quasiparticle hydrodynamics

A possible way to restore thermodynamic consistency for a system of particle with temperature dependent mass is to introduce additional effective mean field through a bag function $B_0(T)$ in order to account for the interactions giving rise to in-medium masses. Then the equilibrium energy-momentum tensor is

$$T_0^{\mu\nu} = \int dP\, p^\mu p^\nu\, f_0 \,+ B_0(T)\, g^{\mu\nu}, \tag{1}$$

where $g^{\mu\nu}$ is the Minkowski metric tensor, p^μ is the quasiparticle four-momentum and dP is the invariant momentum integration measure. Here $f_0 = g\exp[-\beta(u\cdot p)]$ is the equilibrium distribution function where g is the degeneracy factor, $\beta = 1/T$ is the inverse temperature and $u\cdot p \equiv u_\mu p^\mu$. The energy density, $\mathcal{E}_0 = u_\alpha u_\beta T_0^{\alpha\beta}$, and pressure, $\mathcal{P}_0 = -\frac{1}{3}\Delta_{\alpha\beta}T_0^{\alpha\beta}$, are guaranteed to satisfy the thermodynamic relation $d\mathcal{P}_0/dT = (\mathcal{E}_0+\mathcal{P}_0)/T$ if

$$dB_0 + m\,dm\int dP\, f_0 = 0, \tag{2}$$

where $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$, and we also define $\Delta^{\mu\nu}_{\alpha\beta} \equiv \frac{1}{2}(\Delta^\mu_\alpha\Delta^\nu_\beta + \Delta^\mu_\beta\Delta^\nu_\alpha - \frac{2}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta})$.

For the non-equilibrium case, we propose the energy-momentum tensor of the form [2]

$$T^{\mu\nu} = \int dP\, p^\mu p^\nu\, f \,+ B^{\mu\nu}, \tag{3}$$

where f is the non-equilibrium distribution function and

$$B^{\mu\nu} = B_0 g^{\mu\nu} + \delta B^{\mu\nu}. \tag{4}$$

The viscous evolution of a system is governed by the equation $\partial_\mu T^{\mu\nu} = 0$. Next, we derive evolution equations for bulk viscous pressure, Π, and shear stress tensor, $\pi^{\mu\nu}$, defined as

$$\Pi \equiv -\frac{1}{3}\Delta_{\alpha\beta}\delta T^{\alpha\beta}, \quad \pi^{\mu\nu} \equiv \Delta^{\mu\nu}_{\alpha\beta}\delta T^{\alpha\beta}, \tag{5}$$

where $\delta T^{\alpha\beta} \equiv T^{\alpha\beta} - T_0^{\alpha\beta}$. To that end, we consider Boltzmann equation for temperature-dependent particle masses in the relaxation-time approximation

$$p^\mu\partial_\mu f + m\,(\partial^\rho m)\,\partial^{(p)}_\rho f = -\frac{(u\cdot p)}{\tau_R}\delta f, \tag{6}$$

where $\delta f \equiv f - f_0$ and τ_R is relaxation time.

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Dissipative evolution equations

In order to derive the evolution equations for the dissipative quantities, we start with the Chapman-Enskog-like iterative solution of the Boltzmann equation, Eq. (6). The first-order solution is given by [3, 4]

$$\delta f_1 = -\frac{\tau_R}{(u\cdot p)}\left[(p\cdot\partial)f_0 + m\,(\partial_\rho m)\,\partial^\rho_{(p)} f_0\right].$$

Up to first-order in gradients, the above equation can be written as

$$\delta f_1 = \frac{f_0\,\beta\tau_R}{(u\cdot p)}\left[\left(\frac{1}{3}p^2 - m^2\kappa\, c_s^2\right) - \left(\frac{1}{3} - c_s^2\right) \times (u\cdot p)^2\right]\theta + \frac{f_0\,\beta\tau_R}{(u\cdot p)}(p\cdot\sigma\cdot p), \qquad (7)$$

where $\kappa \equiv (T/m)(dm/dT)$ and c_s^2 is the squared speed of sound.

We can now proceed to obtain the first-order equations for the dissipative quantities. Using Eq. (7) to substitute $f = f_0 + \delta f_1$ in Eqs. (3) and keeping terms which are first-order in gradients in Eq. (5), we get

$$\Pi = -\beta_\Pi\,\tau_R\,\theta, \quad \pi^{\mu\nu} = 2\,\beta_\pi\tau_R\,\sigma^{\mu\nu}, \qquad (8)$$

where $\beta_\pi = \beta I_{3,2}$ and

$$\beta_\Pi = \frac{5}{3}\,\beta\, I_{3,2} - c_s^2\,(\mathcal{E}+\mathcal{P}) + \kappa c_s^2\, m^2\beta\, I_{1,1}\ .$$

In the above equations, the integral coefficients $I_{n,q}$ are defined as

$$I_{n,q} \equiv \frac{(-1)^q}{(2q+1)!!}\int dP\,(u\cdot p)^{n-2q}\,(p\cdot\Delta\cdot p)^q\, f_0.$$

The coefficient of bulk and shear viscosities are given by $\zeta = \beta_\Pi\tau_R$ and $\eta = \beta_\pi\tau_R$, respectively.

In order to obtain the second-order evolution equation for dissipative quantities, we follow the procedure given in Refs. [3, 4]. Using Eq. (7) in Eqs. (3), keeping terms which are second-order in gradients in Eq. (5), and performing the integrations, we get [2]

$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} - \beta_\Pi\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi_{\alpha\beta}\sigma^{\alpha\beta}, \quad (9)$$

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + 2\beta_\pi\sigma^{\mu\nu} - \tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}, \qquad (10)$$

where $\omega^{\mu\nu} \equiv \frac{1}{2}(\nabla^\mu u^\nu - \nabla^\nu u^\mu)$ is the vorticity tensor. The above equations are the main results of the present work. It is interesting to note that, up to second-order, the above evolution equations for bulk and shear pressure do not have contributions from non-equilibrium bag correction, i.e., $\delta B^{\mu\nu}$ in Eq. (4).

The second-order transport coefficients in Eqs. (9) and (10) are obtained as

$$\delta_{\Pi\Pi} = -\frac{5}{9}\chi - \left(1 - \kappa m^2\frac{I_{1,1}}{I_{3,1}}\right)c_s^2 + \frac{1}{3}\frac{\beta\kappa c_s^2 m^2}{\beta_\Pi}\left[\left(1-3c_s^2\right)\left(\beta I_{2,1} - I_{1,1}\right) - \left(1-3\kappa c_s^2\right)m^2\left(\beta I_{0,1} + I_{-1,1}\right)\right],$$

$$\lambda_{\Pi\pi} = \frac{\beta}{3\beta_\pi}(2I_{3,2} - 7I_{3,3}) - \left(1 - \kappa m^2\frac{I_{1,1}}{I_{3,1}}\right)c_s^2,$$

$$\delta_{\pi\pi} = \frac{5}{3} - \frac{7}{3}\frac{\beta}{\beta_\pi}I_{3,3} - \frac{\beta}{\beta_\pi}\kappa c_s^2 m^2\left(I_{1,2} - I_{1,1}\right),$$

$$\tau_{\pi\pi} = 2 - \frac{4\beta}{\beta_\pi}I_{3,3}, \quad \lambda_{\pi\Pi} = -\frac{2}{3}\chi,$$

where $\chi = \frac{\beta}{\beta_\Pi}[\left(1-3c_s^2\right)\left(I_{3,2} - I_{3,1}\right) - \left(1-3\kappa c_s^2\right)m^2\left(I_{1,2} - I_{1,1}\right)]$. As expected, in the constant mass limit, $\kappa \to 0$, the above transport coefficients match exactly with those obtained in Ref. [5]. The integral coefficients, $I_{n,q}$, can be obtained in terms of modified Bessel functions of the second kind, $K_n(z)$ where $z = m(T)/T$. A few of them are given below [2, 5]:

$$I_{2,1} = \frac{gT^4z^2}{2\pi^2}K_2(z), \quad I_{3,1} = \frac{gT^5z^3}{2\pi^2}K_3(z).$$

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