Cornell Potential Model for Strongly Coupled Quark Gluon Plasma

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Introduction

Experiments carried out at RHIC have shown that Quark Gluon Plasma(QGP) is strongly coupled at temperatures below $3T_c$ [1]. This work is an extension of the work we reported earlier [2]. Here we study sQGP using Cornell Potential by considering QGP as consisting of bound states of quarks and gluons. We will compare the equation of state (EOS) thus obtained with lattice data. We proceed by using an approximate solution of Schroedinger equation with Cornell potential and using it to evaluate the second Virial Coefficient quantum mechanically.

Approximate solution for quark-antiquark pair in Cornell Potential

The Cornell Potential between quarks and anti-quarks is given by,

$$V(r) = ar - \frac{b}{r}. \tag{1}$$

The Schroedinger equation with this potential does not admit exact solutions. So one has to resort to approximation methods. One such method that is useful is known as the Nikiferov-Uvarov method. We make use of the following solution [3].

$$E_{nl} = \frac{3a}{\delta} - \frac{2\mu\left(b + \frac{3a}{\delta^2}\right)^2}{\left[(2n+1) \pm \sqrt{1 + 4l(l+1) + \frac{8\mu a}{\delta^3}}\right]^2}, \tag{2}$$

where, δ is a parameter equal to the inverse of what is referred to as the characteristic radius r_0. This solution is shown to reproduce the mass spectra of heavy mesons successfully with suitable choices of the parameters a, b, and δ. n takes the values 0, 1, 2, etc.

Quantum Theory of the second Virial Coefficient

Pais and Uhlenbeck have obtained an exact quantum mechanical expression for the second Virial Coefficient[4]. We use it in the form of given in[5].

$$\begin{aligned} b_2 - b_2^0 &= 8^{1/2} \sum_B e^{-\beta \epsilon_B} \\ &+ \frac{8^{1/2}}{\pi} \sum_l{}' (2l+1) \int_0^\infty dk \Big[e^{-\beta \hbar^2 k^2/m} \\ &\times \frac{\partial \eta_l(K)}{\partial k} \Big], \end{aligned}$$

where, b_2, b_2^0 are the second cluster integrals for the interacting and non-interacting cases respectively. ϵ_B are the bound states and η_l is the phase shift during scattering. Since we deal only with the bound states, we can take the phase shifts to be equal to zero[4] . Thus for our case,

$$b_2 - b_2^0 = 8^{1/2} \sum_B e^{-\beta \epsilon_B}, \tag{3}$$

where, the summation goes over all bound states made possible by the two body interaction. The second Virial coefficient is related to the second cluster integral by,

$$b_2 = -a_2. \tag{4}$$

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The equation of state for sQGP

Once the second cluster integral is obtained, we can construct the equation of state of the system using,

$$\frac{P}{KT} = \frac{1}{\lambda^3} \sum_{l=1}^{\infty} b_l z^l, \tag{5}$$

where, z is the fugacity which is taken equal to 1. We shall take up to the second term in the above infinite sum. Thus,

$$\frac{P}{KT} = \frac{1}{\lambda^3}[b_1 + b_2]. \tag{6}$$

$b_1 = 1$, and b_2 can be evaluated by using equation (2) in equation(3).

Using the above we first obtain the EOS for the pure gauge case . We find a reasonably good fit with, $a = 0.155$, $\delta = 0.380$ and the summation carried out upto $n = 1360$ corresponding to a gluon mass of 0.0002 GeV. The EOS thus obtained is plotted and compared with lattice data [6]. in the lowest curve in FIG.1.

For the 2-flavor casse, the total pressure is found by adding the separate contributions from gluons as well as quarks. The mass of the quarks are taken to be $m = 0.0016$ GeV and correspondingly the summation is carried over upto $n = 520$. The other parameters are chosen as $\delta = 0.690$, $a = 0.134$, $b = 0$. The EOS thus obtained is compared with lattice data for the 2 flavor QGP in the second curve from the bottom in FIG1.

We proceed along the same way for 3-flavor case and obtain a good fit with lattice data for $m = 0.0016GeV$, $\delta = 0.840$, $a = 0.138$, $b = 0$. The summation has been carried over up to $n = 385$.

Results and discussions

In this work we have obtained an equation of state(EOS) of QGP, assuming it to be composed of bound states of quarks and gluons. The EOS thus obtained shows a reasonable fit with the lattice data up to around $3T_c$ with $T_c = .160GeV$. Our EOS starts to deviate from the lattice data around $3T_c$, suggesting that the free quarks and gluons start to dominate. The EOS does not depend on the value of the parameter b for a large range of values including $b = 0$. This is because of the very low masses involved. The upper limit on summation over n is believed to be due to the screening effect which we have not included in the potential. In future work we will seek to improve the result by including the effects of both the boundstates and non-bound states.

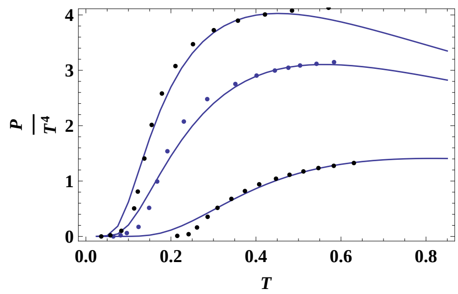

FIG. 1: Plots of P/T^4 as a function of T(in GeV) using the Cornell potential, and lattice result for pure gauge, 2 flavour and 3 flavor QGP

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References

[1] E. Shuryak, arXiv:hep-ph/0312227v2 (2003).

[2] S. Koothottil, J. P. Prasanth, and V. M. Bannur, DAE Symp. Nucl. Phys. **61**, 814 (2016).

[3] S. M. Kuchin and N. V. Maksimenko, Universal Journal of Physics and Application 1 **3**, 295 (2013).

[4] A. Pais and G. E. Uhlenbeck, Physical Review **116** (1959).

[5] R. K. Pathria and P. D. Beale, *Statistical Mechanics* (Butterworth-Heinemann, 2011).

[6] E. Laermann and O. Philipsen, arXiv:hep-ph/0303042v1 (2003).