Two-body matrix elements calculations of neutrinoless double beta decay of $^{76}_{32}Ge$ using nuclear shell model

Shahariar Sarkar,* Pawan Kumar, Kanhaiya Jha, and P.K. Raina
Department of Physics, Indian Institute of Technology Ropar, Rupnagar, Punjab-140001, INDIA

Introduction

Neutrinoless double beta decay is a rare lepton number violating weak process where two neutrons inside some even even nuclei converted into two protons and two electrons with neutrino as a virtual state[1]. It proves neutrinos are Majorana particles rather than Dirac particles and also predict absolute neutrino mass. But even after 70 years of its prediction, it is still unobserved. Limitations of understanding complex nuclear structure leads to uncertainties in the calculations of two-body matrix elements(TBME) which leads to uncertainties in the calculations of final nuclear matrix elements and thus uncertainties in predicting proper half life. In this paper we will discuss those theoretical uncertainties to calculate two-body matrix elements(TBME) for neutrinoless double beta decay $^{76}_{32}Ge \rightarrow^{76}_{34} Se + e^- + e^-$ using nuclear shell model.

Theoretical Framework

Here we did calculations for light Majorana neutrino exchange mechanism.

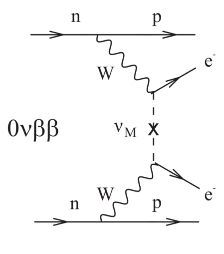

FIG. 1: Light neutrino exchange mechanism of neutrinoless double beta decay

*Electronic address: shahariar.sarkar@iitrpr.ac.in

Decay rate for this process can be written as

$$\Gamma^{0\nu} = \frac{1}{T^{0\nu}_{1/2}} = G^{0\nu}(Q,Z)|m_{\beta\beta}|^2|M^{0\nu}|^2 \quad (1)$$

$G^{0\nu}(Q,Z)$ is phase space factor and $m_{\beta\beta}$ is absolute mass of Majorana neutrino. $M^{0\nu}$ is nuclear matrix element given by

$$M^{0\nu} = M^{0\nu}_{GT} - \frac{g_V^2}{g_A^2}M^{0\nu}_F + M^{0\nu}_T \quad (2)$$

Each part of nuclear matrix element can be written as [2]

$$M^{0\nu}_\alpha = \sum_{\substack{j_{p1},j_{p2}\\ j_{n1},j_{n2},J}} TBTD(j_{p1}j_{p2}j_{n1}j_{n2}J^\pi_i J^\pi_f J) \times TBME((j_{p1},j_{p2},j_{n1},j_{n2},J,J^\pi_i,J_f) \quad (3)$$

Here Two Body Matrix Elements $TBME_\alpha((j_{p1},j_{p2},j_{n1},j_{n2},JT,J^\pi_i,J_f) =< j_{p1}j_{p2},JT,J^+_i|\tau^+_1\tau^+_2 O^{0\nu}_\alpha|j_{n1}j_{n2},JT,J^+_f >_A$ $\alpha = F,GT,T$ and A stands for anti-symmetric. Transition operators $O^{0\nu}_\alpha$ are written as $O^{0\nu}_{GT} = \vec{\sigma}_1.\vec{\sigma}_2 H_{GT}(r)$, $O^{0\nu}_F = H_F(r)$, $O^{0\nu}_T = (3(\vec{\sigma}_1.\hat{r})(\vec{\sigma}_2.\hat{r}) - \vec{\sigma}_1.\vec{\sigma}_2)H_T(r)$. For calculation of TBME we use Talmi Moshinisky transformations to transform problem from individual coordinate system of nucleons to center of mass(c.o.m) and relative coordinate system. Then we need to calculate neutrino potential integral of the form $NP^\alpha(n,l) =< nl|H_\alpha(r)|nl >= \int_0^\infty R_{nl}R_{nl}r^2 * H_\alpha(r)dr$. Here R_{nl} is 3D harmonic oscillator radial wave function. r is inter nucleon distance. $H_\alpha(r)$ is neutrino potential and written as a integral of neutrino momentum q [2]
$H_\alpha(r) = \frac{2R}{\pi}\int_0^\infty \frac{q\times j_{0,2}(qr)\times g^\alpha(q)dq}{q+\overline{E}}$

It is the most important part of TBME calculations containing various sources of uncertainties.One such uncertainty source is finite nucleon size. Nucleons are made up of quarks and it is taken care by a form factor $f_{FNS}(q^2) = \frac{1}{(1+\frac{q^2}{M_A})^2}$. In addition to $V - A$ coupling there are higher order coupling like weak magnetism and pseudoscalar at nucleon level decay and are taken care by higher order coupling term[2]

$$g^F(q) = g_V^2(q^2) \quad (4)$$

$$g^{GT}(q) = \frac{g_A^2(q^2)}{g_A^2}\left[1 - \frac{2}{3}\frac{q^2}{q^2+m_\pi^2} + \frac{1}{3}\frac{q^4}{(q^2+m_\pi^2)^2}\right] + \frac{2}{3}\left(\frac{g_M(q^2)}{g_A}\right)^2 \times \frac{q^2}{4m_p^2} \quad (5)$$

$$g^T(q) = \frac{g_A^2(q^2)}{g_A^2}\left[\frac{2}{3}\frac{q^2}{q^2+m_\pi^2} - \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right] + \frac{1}{3}\left(\frac{g_M(q^2)}{g_A)}\right)^2 \frac{q^2}{4m_p^2} \quad (6)$$

Another uncertainty term is short range correlation. Force between nucleons at very short distance is strongly repulsive. It is taken care by Jastrow function $R_{nl} \to f_{Jastrow}R_{nl}$, where $f_{Jastrow}(r) = 1 - ce^{-ar^2}(1 - br^2)$ $a = 1.10, b = 0.68, c = 1.00$ are constant parameter for our calculations.

Calculations of Two-Body Matrix Elements

We did calculations using nuclear shell model for neutrinoless double beta decay of

$$^{76}_{32}Ge \to ^{76}_{34} Se + e^- + e^-.$$

Active nucleons of $^{76}_{32}Ge$ and $^{76}_{34}Se$ are in $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, $g_{9/2}$ orbits. Using above formalism, important part neutrino potential integral for both without SRC and with SRC were calculated and few results are shown in table I. Tensor parts are negligible.

Calculation of Talmi Moshinisky brackets were done using the formalism given in [3]. Spin parts of TBMEs were calculated using standard method.Then TBMEs were calculated without and with SRC and few results are given in table II and III.Calculations for two body transition densities(TBTD)will be presented in the symposium which is necessary for final NME calculations[4].

TABLE I: Table of calculated values of neutrino potential integral without and with SRC

n, l	NP^F	NP^F_{SRC}	NP^{GT}	NP^{GT}_{SRC}
0,0	0.02365	0.00994	0.01518	0.00637
3,0	0.03163	0.00571	0.02031	0.00366
2,1	0.00559	0.00421	0.00358	0.00270
1,1	0.00488	0.00409	0.00313	0.00262

TABLE II: Table of calculated Fermi part of TBME without and with SRC

$j_{p1}, j_{p2}, j_{n1}, j_{n2}, J^\pi$	$TBME^F$	$TBME^F_{SRC}$
1/2,1/2,3/2,3/2,0$^+$	0.00736	0.00237
9/2,9/2,9/2,9/2,4$^+$	0.00209	0.00088
3/2,3/2,5/2,5/2,0$^+$	0.00427	0.00096
9/2,9/2,9/2,9/2,0$^+$	0.01242	0.00309

TABLE III: Table of calculated GT part of TBME without and with SRC

$j_{p1}, j_{p2}, j_{n1}, j_{n2}, J^\pi$	$TBME^{GT}$	$TBME^{GT}_{SRC}$
1/2,1/2,3/2,3/2,0$^+$	-0.01417	-0.00455
9/2,9/2,9/2,9/2,4$^+$	-0.00402	-0.00169
3/2,3/2,5/2,5/2,0$^+$	-0.00822	-0.00185
9/2,9/2,9/2,9/2,0$^+$	-0.02394	-0.00593

Summary

We have calculated two-body matrix elements(TBME) of nuclear matrix elements(NME) for the decay $^{76}_{32}Ge \to^{76}_{34} Se + e^- + e^-$ which is most important to predict proper half life. We discussed various sources of uncertainties in the calculations.

Acknowledgments

We thanks MHRD, Govt. of India for providing funds and IIT Ropar.

References

[1] W. Furry. Phys. Rev., 56:1184, 1939
[2] Mihai Horoi1,and Sabin Stoica Phys. Rev. C 81, 024321 (2010)
[3] G. Kamuntavi et al. Nuclear Physics A 695 (2001) 191201).
[4] B.A. Brown, M. Horoi, and R.A. Senkov PRL 113, 262501 (2014) (2014).