Modification of pairing energy term in the liquid drop model

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Introduction

Pairing correlations play an important role in describing the low-energy nuclear structure, reaction cross-sections as well as fission and fusion processes. Pairing energy term in the mean-field models takes into account the tendency of like nucleons to form pairs in analogy with Cooper pairs in metallic superconductors [1]. The present approach aims towards the modification of pairing energy term in the liquid drop model (LDM) using semiclassical level density in the Bardeen-Cooper-Schrieffer (BCS) theory. In this work, the semiclassical trace formula for axially symmetric harmonic oscillator potential along with spin-orbit interactions [2] is employed to study the effects of pairing in prolate deformed nuclear systems such as Te^{106}, Te^{108}, Te^{110}.

Redefining the pairing energy term using BCS theory

The liquid drop model is the first ever nuclear model, proposed by George Gamow, in which the nucleus is treated as a charged drop of liquid and hence the nucleus can be described in terms of volume, surface tension, density etc., modeled as the famous mass formula of Bethe and von Weizsäcker:

$$\begin{aligned} B(A,Z) &= a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} \\ &- a_a \frac{(N-Z)^2}{A} + \delta(A). \end{aligned} \quad (1)$$

The last term, known as "pairing energy", takes care of the tendency of like nucleons to pair up, thus resulting in an increased binding energy of the system. The coefficients a_i in the LDM are given as [3], a_v=15.753 MeV, a_s=17.804 MeV, a_c=0.7103 MeV, a_a=23.69 MeV and the pairing energy term is defined as:

$$\delta(A)=\begin{cases} 33.6\ A^{-3/4} & \text{for e-e nuclei} \\ -33.6\ A^{-3/4} & \text{for o-o nuclei} \\ 0 & \text{for e-o nuclei} \end{cases}$$

The above term can be modified using BCS equations [4] at zero temperature for protons and neutrons:

$$\begin{aligned} N_{n,p} &= 2\int_{-\infty}^{r} \tilde{g}_{n,p}(E)dE \\ &+ \int_{r}^{s} \tilde{g}_{n,p}(E)\left[1-\frac{E-\lambda'_{n,p}}{E_q^{n,p}}dE\right] \\ \frac{2}{G_{n,p}} &= \int_{r}^{s} \frac{\tilde{g}_{n,p}(E)}{E_q^{n,p}}dE \end{aligned} \quad (2)$$

and the pairing correlation energy δP is defined as the difference between the energies with and without pairing effects, i.e. $\delta P_{n,p} = |\tilde{E}_{pair} - \tilde{E}|$ where

$$\begin{aligned} \tilde{E}_{pair}(n,p) &= 2\int_{-\infty}^{r} E\tilde{g}_{n,p}(E)dE - \frac{\Delta_{n,p}^2}{G_{n,p}} \\ &+ \int_{r}^{s} E\tilde{g}_{n,p}(E)\left[1-\frac{E-\lambda'_{n,p}}{E_q^{n,p}}\right]dE \\ \tilde{E}(n,p) = &= 2\int_{-\infty}^{E_F^{n,p}} E\tilde{g}_{n,p}(E)dE. \end{aligned} \quad (3)$$

Here, $r,s = \lambda'_{n,p} \mp \hbar\omega_0^0(n,p)$. $\lambda'_{n,p}$ and $E_F^{n,p}$ are the chemical potentials with and without pairing. The quasiparticle energy is defined as, $E_q = ((E-\lambda'_{n,p})^2+\Delta^2_{n,p})^{1/2}$. $\Delta_{n,p}$ are the odd-even mass differences obtained from the four-point formula and the spacing between the oscillator levels is chosen as:

$$\hbar\omega_0^0(n,p) = \frac{41}{A^{\frac{1}{3}}}\left(1 \pm \frac{N-Z}{A}\right)^{\frac{1}{3}}.$$

The semiclassical expression of the average level density for axially symmetric harmonic oscillator potential with spin-orbit interactions is given as [2]:

$$\begin{aligned} \tilde{g}(E) &= \frac{E^2}{2a\hbar\omega_0^{0\,3}}\left[1+\kappa^2 b\right] \\ &- \frac{b}{24a\hbar\omega_0^0}\left[1+\kappa^2\frac{\left(b^2+2c\right)}{b}\right] \\ &+ \frac{E\kappa^3 c}{3a\hbar\omega_0^{0\,2}} + \mathcal{O}(\hbar^4\kappa^4) + ... \end{aligned} \quad (4)$$

where, $a = \hbar\omega_\perp{}^2\hbar\omega_z, b = 2\hbar\omega_\perp{}^2 + \hbar\omega_z{}^2, c = \hbar\omega_\perp{}^4 + 2\hbar\omega_\perp{}^2\hbar\omega_z{}^2$. The oscillator frequencies are defined in terms of the deformation parameter ϵ as:

$$\omega_\perp = \omega_0(\epsilon)\left(1+\frac{1}{3}\epsilon\right), \ \ \omega_z = \omega_0(\epsilon)\left(1-\frac{2}{3}\epsilon\right)$$

$\omega_0(\epsilon)$ is determined from the condition of incompressibility of nuclear matter and is given as:

$$\omega_0(\epsilon) = \omega_0^0\left(1+\frac{\epsilon^2}{9}\right) \quad (5)$$

The Nilsson deformation parameter ϵ is defined in terms of quadrupole deformation β_2 as, $\epsilon \approx 0.95\beta_2$.

Results and Discussions

Despite its simplicity, the liquid drop model can describe the fission, fusion and α decay potential barriers quite well. Moreover LDM with little modifications can serve as a good tool in the study of binding energies of new nuclides (superheavy elements, drip line nuclei). These modifications can be in terms of shell structures and pairing effects. The present approach highlights the modification of pairing correlations using semiclassical-BCS theory. The differences in the evaluated and the experimental binding energies may be resolved by considering shell effects. To the discerning reader who notes the smallness of the difference between $B(A,Z)$ and $B_M(A,Z)$, we would like to emphasize that this small change is towards the desirable correct value.

Nucleus	κ_p	κ_n	Δ_p (MeV)	Δ_n (MeV)	β_2
Te^{106}	-0.065	-0.070	0.94	1.01	0.119
Te^{108}	-0.065	-0.070	1.02	1.25	0.139
Te^{110}	-0.065	-0.070	1.01	1.15	0.150

Table 1: Spin-orbit strength parameters $\kappa_{p,n}$, pairing gaps $\Delta_{p,n}$ and quadrupole deformation β_2 [5] used for our calculations.

Nucleus	$\delta(A)$ (MeV)	δP (MeV)	$B(A,Z)$ (MeV)	$B_M(A,Z)$ (MeV)	$B_E(A,Z)$ (MeV)
Te^{106}	1.017	1.626	865.34	865.95	873.10
Te^{108}	1.003	2.295	891.74	893.03	896.80
Te^{110}	0.989	2.091	916.48	917.58	919.39

Table 2: Values of modified pairing energy term, $\delta P = \delta P_p + \delta P_n$ and the corresponding modified B_M(A,Z) in comparison to LDM $\delta(A)$ and B(A,Z), the expt.value B_E(A,Z) [6].

References

[1] A. Bohr, B.R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958).

[2] M. Brack, S.R. Jain, Physical Review A **51**, 3462 (1995); B.K. Jennings, R.K. Bhaduri, M. Brack, Nuclear Physics A **253**, 29 (1975).

[3] J.L. Basdevant, J. Rich, M. Spiro, Fundamentals in Nuclear Physics. Springer (2005).

[4] A.S. Jensen, Jens Damgaard, Nuclear Physics A **203**, 578 (1973).

[5] http://t2.lanl.gov/nis/molleretal.

[6] https://www.nndc.bnl.gov.