

Semiclassical calculation of pairing correlations in deformed nuclei

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Introduction

The periodic orbit theory (POT) developed by Gutzwiller is a semiclassical approach in which the density of the energy levels of a quantum system are approximated in terms of the periodic orbits of the corresponding classical system through the trace formula [1]. Shell structures, pairing correlations and deformation effects play an important role in the understanding of low-energy nuclear structure as well as reaction dynamics. In this work, the semiclassical trace formula for axially symmetric harmonic oscillator potential along with spin-orbit interactions is utilized to study the effects of pairing in Dy¹⁶⁰ (prolate system) using finite temperature Bardeen, Cooper and Schrieffer theory (FT-BCS).

Axially symmetric harmonic oscillator

For the understanding of deformed nuclear systems, Nilsson model [2] was proposed. It is basically the generalization of the isotropic three dimensional harmonic oscillator potential with axial symmetry. The Nilsson single-particle Hamiltonian without the centrifugal term can be written as:

$$H_N = \frac{-\hbar^2}{2m} \nabla^2 + \frac{1}{2} m [\omega_{\perp}^2 (x^2 + y^2) + \omega_z^2 z^2 + C \mathbf{l} \cdot \mathbf{s}] \quad (1)$$

where, C is defined in terms of spin-orbit strength parameter κ , $C = 2\kappa\hbar\omega_0^0$. The spacing between the oscillator levels is chosen as:

$$\hbar\omega_0^0(n, p) = \frac{41}{A^{\frac{1}{3}}} \left(1 \pm \frac{N-Z}{A} \right)^{\frac{1}{3}}. \quad (2)$$

The oscillator frequencies are defined in terms of the deformation parameter ϵ as:

$$\omega_{\perp} = \omega_0(\epsilon) \left(1 + \frac{1}{3}\epsilon \right), \quad \omega_z = \omega_0(\epsilon) \left(1 - \frac{2}{3}\epsilon \right)$$

$\omega_0(\epsilon)$ is determined from the condition of incompressibility of nuclear matter and is given as:

$$\omega_0(\epsilon) = \omega_0^0 \left(1 + \frac{\epsilon^2}{9} \right) \quad (3)$$

The Nilsson deformation parameter ϵ is defined in terms of quadrupole deformation β_2 as, $\epsilon \approx 0.95\beta_2$. The semiclassical expression of the average level density for axially symmetric harmonic oscillator potential is given as [1]:

$$\tilde{g}(E) = \frac{1}{2\hbar\omega_{\perp}\hbar\omega_z} \left[E^2 - \frac{1}{12}(2\hbar\omega_{\perp}^2 + \hbar\omega_z^2) \right] \quad (4)$$

Using the methodology of Jennings et al.[3], the inclusion of spin orbit interactions to above expression yields:

$$\begin{aligned} \tilde{g}(E) &= \frac{E^2}{2a\hbar\omega_0^0} \left[1 + \kappa^2 b \right] - \frac{b}{24a\hbar\omega_0^0} \left[1 + \kappa^2 \frac{(b^2 + 2c)}{b} \right] \\ &+ \frac{E\kappa^3 c}{3a\hbar\omega_0^0} + \mathcal{O}(\hbar^4 \kappa^4) + \dots \end{aligned} \quad (5)$$

where, $a = \hbar\omega_{\perp}^2 \hbar\omega_z$, $b = 2\hbar\omega_{\perp}^2 + \hbar\omega_z^2$, $c = \hbar\omega_{\perp}^4 + 2\hbar\omega_{\perp}^2 \hbar\omega_z^2$.

Pairing correlations in Dy¹⁶⁰

It is believed that nuclei consist of nucleonic Cooper pairs at energies close to Fermi energy. Dy¹⁶⁰ is an even-even nucleus with quadrupole deformation $\beta_2 = 0.3343$ [4]. We have studied thermodynamic properties such as excitation

energy, entropy and nuclear level density in this nucleus using FT-BCS theory [5]:

$$\begin{aligned}
 U &= \sum_k \epsilon_k \left(1 - \frac{\epsilon_k - \lambda}{E_k} \tanh \left(\frac{\beta E_k}{2} \right) \right) \\
 S &= 2 \sum_k \log_e (1 + \exp(-\beta E_k)) \\
 &+ 2\beta \sum_k \frac{E_k}{1 + \exp(\beta E_k)}
 \end{aligned} \tag{6}$$

in which the expression of level density $\tilde{g}(E)$ as given in eq. (5) is employed. Here \sum_k represents the sum over single particle states and E_k represents the quasiparticle energy.

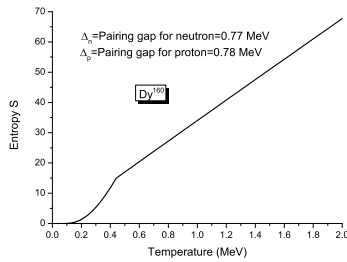


Figure 1: Excitation energy as a function of temperature for Dy¹⁶⁰. Superfluid-normal phase transition is indicated by two arrows.

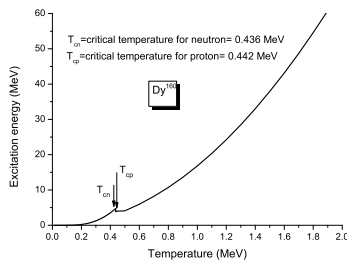


Figure 2: Entropy as a function of temperature for Dy¹⁶⁰. It is clearly seen that the system is less random below critical temperatures.

The nuclear level density is evaluated in the saddle point approximation, $\rho = \frac{\exp S}{(2\pi)^{3/2} \sqrt{D}}$,

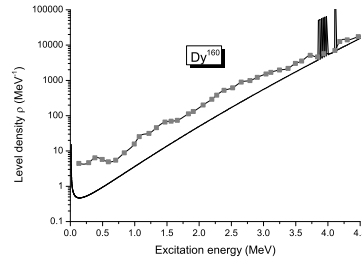


Figure 3: Level density as a function of excitation energy for Dy¹⁶⁰. The solid grey squares represent the experimental data and the solid line represents the theoretical result obtained.

where D is the determinant of the 3×3 matrix [5]. The odd-even mass differences are evaluated using four-point formula [2]. Above the critical temperatures, we have followed the methodology as described in [6]. The results of E, S and ρ for Dy¹⁶⁰ are shown in figs. 1, 2 and 3 respectively. In Fig. 3. the peaks in the level density represent the pair breaking process. Overall the agreement with experimental data [7] is good.

References

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