Description of superdeformed band in even-even $A < 100$ mass region nuclei

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Introduction

The most important way of assigning spin in superdeformed bands are theoretical methods [1] in terms of gamma energies [2] are the only available information for the SD bands. Several theoretical methods e.g. band energy expression $J(J+1)$ [3] energy power series in term of angular momentum and the super symmetric algebraic model including many body interaction (SAM) [4] shows the beneficial results.
These techniques implicate direct and indirect methods or allocating the spin to the state in super deformed bands. In this process the spin may be manifested as an expansion of the rotational frequency.
In our present study, we put forward variable moment of inertia (VMI) equation, *ab* formula and Harries (ω^2) expanssion to determine the band head spin. In VMI equation band head spin is directly obtained from ratio between experimental transition energies. The ratio between γ energies, R, derived from the VMI equation depends upon restoring constant (C), moment of inertia (I) and unknown band head spin (I_0). The another simple and effective method to determine the band head spin values of the superdeformed band is *ab*-method [5].

VMI Model

Variable moment of inertia model was first proposed by Mariscotti et al. [6] to predict different level energies of ground state bands in even-even nuclei. This method expresses the rotational energies as an expansion in terms of $I(I+1)$ and generalizes the ratio of higher mass nucleus very efficiently. VMI equation for rotational bands is given by the sum of potential energy term and rotational energy term,

$$E_I = E_0 + \frac{I(I+1) - I_0(I_0+1)}{2J_I} + \frac{C(J_I - J_0)^2}{2} \qquad (1)$$

where J_0 is band head moment of inertia, C is restoring force constant, E_0 is energy of band head, I is angular momentum and J_I is corresponding moment of inertia of the state with angular momentum I. Second term represents rotational energy term and third term represents the potential energy term. By using equilibrium condition we can write

$$\frac{\partial E(J_I)}{\partial J_I} = 0 \qquad (2)$$

$$-\frac{I(I+1) - I_0(I_0+1)}{2J_I^2} + CJ_I - CJ_0 = 0 \qquad (3)$$

$$J_I^3 - J_I^2 J_0 - \frac{I(I+1) - I_0(I_0+1)}{2C} = 0 \qquad (4)$$

By using Eq. 3, we can rearrange Eq. 1 as:

$$\begin{aligned} E_I =& E_0 + \left[\frac{I(I+1) - I_0(I_0+1)}{2J_0}\right] \\ &\times \left[1 + \frac{I(I+1) - I_0(I_0+1)}{4CJ_0^3}\right] \end{aligned} \qquad (5)$$

For a superdeformed cascade $I_0 + 2n \to I_0 + 2n - 2 \to .....I_0 + 2 \to I_0$ the transition energy is expressed as

$$E_\gamma(I \to I-2) = E(I) - E(I-2) \qquad (6)$$

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$$E_\gamma(I \to I-2) = \frac{[I(I+1)-(I-2)(I-1)]}{2J_0} + \frac{[I(I+1)]^2 - [(I-2)(I-1)]^2}{8CJ_0^4} \quad (7)$$

The observed transition energies are fitted to the Eq.7 and then values of parameters J_0 and C are obtained.

Harris ω^2 expansion

Harris suggested that the rotational energy of deformed nuclei can be represented in powers of ω^2. We have used the expansion with three parameters,

$$E(\omega) = \alpha\omega^2 + \beta\omega^4 + \gamma\omega^6 \quad (8)$$

Using this equation dynamic moment of inertia ($\theta^{(2)}$) is given as

$$\theta^{(2)} = A + B\omega^2 + C\omega^4 \quad (9)$$

Where $A = 2\alpha; B = 4\beta$ and $C = 6\gamma$. Integration of above equation with respect to ω will lead us to the equation of spin as a function of ω,

$$\hat{I} = A\omega + \frac{B}{3}\omega^3 + \frac{C}{5}\omega^5 + i_0 \quad (10)$$

Here i_0 is constant of integration called aligned spin and it can take values either half or zero.

Result and Discussion

By using Variable Moment of Inertia (VMI) and Harris (ω^2), band head moment of inertia (J_0) and restoring constant (C) are determined by fitting experimental transition energies for $^{83-84}$Zr(SD-1), ^{86}Zr and ^{89}Tc(SD-1) and ^{91}Tc(SD-1). The band head spin for these isotopes were determined in the form of gamma transition energy ratio R. The calculated pin value for different isotopes is compared with other theoretical results present in literature. The comparison with other theoretical results is given in Table I-II. Band head spin obtained from Harris (ω^2) expansion deviates largely from the experimental value [7] as compared to the band head spin obtained from VMI and *ab* formula.

TABLE I: Fitting parameters and band head spins calculated using VMI model.

SD Band	E_{exp} (I_0)	$E_{cal}(I_0)$ (VMI)	C	J_0 (VMI)	I_0 (VMI)	I_0 [7]
^{83}Zr (SD-2)	1448	1448.69	-1.60E+09	22.608	15	16.5
^{84}Zr (SD-1)	1526	1524.61	1.00E+08	35.120	23.5	21
^{86}Zr (SD-1)	1518	1515.71	1.00E+08	36.665	24.5	23
^{89}Tc (SD-1)	1147	1144.23	1.20E+07	53.572	24.5	17.5
^{91}Tc (SD-1)	1350	1349.45	-1.10E+09	35.360	22.5	25.5

TABLE II: Three parameters and I_0 values calculated using Harris ω^2 expansion method.

SD Band	A	B	C	I_0	I_0 [7]
^{83}Zr(SD-2)	24.77635	-2.79537	2.35677	18.2	16.5
^{84}Zr(SD-1)	38.91252	-19.1938	6.72496	27.5	21
^{86}Zr(SD-1)	42.20454	-22.8761	8.19151	29.4	23
^{89}Tc(SD-1)	47.59144	-42.8739	20.19851	25.2	17.5
^{91}Tc(SD-1)	39.78658	-8.80174	7.62018	26.4	25.5

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