

## J Selection Rule and Reduced Matrix Elements of K-Isomer Decay: $K=6^+$ Isomer Decay of $^{170}\text{Hf}$ to the Ground Band

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### Introduction

The Hf nuclei are known for K isomers occurring at low excitation energies. A number of strongly deformed bands have been observed in  $^{170}\text{Hf}$  [1],  $^{171,172}\text{Hf}$  [2, 3]. It is of interest to study the hafnium nuclei since they are at the beginning of the transition region between the well deformed rare earth nuclei and the doubly-closed spherical nucleus  $^{208}\text{Pb}$ . The quantum degree of freedom, K, which is the angular momentum projection onto the symmetry axis of a deformed nucleus, is usually a good quantum number. Because of the approximate conservation of K, the decay of these high-K states is subject to selection rules, often causing them to be long-lived isomers. The high-K isomers are able to decay to states of lower K only because of very small admixtures in their wave-functions of the lower values of K. Moreover, high-K isomers systematically occur in neutron-rich nuclei with  $A \geq 150$  which typically have deformed prolate shapes. Nuclear K-isomers decay predominantly by electromagnetic processes ( $\gamma$ -decay or internal conversion). To understand the properties of the ground and K isomeric bands of Hafnium and other nuclei one needs a theoretical model which takes into account the residual interaction among the nucleons in a large enough valence space and gives the proper deformed single-particle states and the multi-nucleon configurations for these nuclei. So we have adopted in this work the Deformed Hartree-Fock (DHF) model

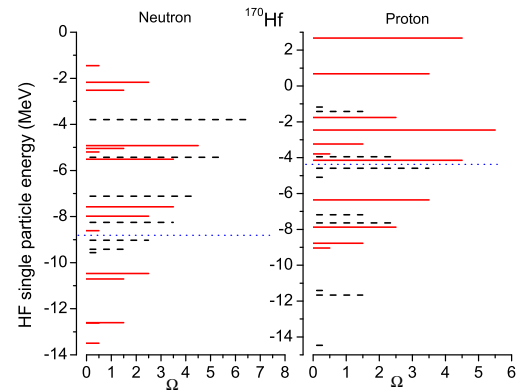


FIG. 1: Deformed HF orbits of  $^{170}\text{Hf}$ .

to get the deformed single-particle states and the deformed multi-nucleon configurations. Ground and K-isomeric intrinsic states are constructed, and for each intrinsic state (configuration), states of good angular momenta are obtained by Angular Momentum Projection. Here we give results for the  $K=6^+$  isomer of  $^{170}\text{Hf}$  and its decay to the ground band.

### Theoretical Framework

A deformed shape such as one described by Slater determinant of deformed orbits  $|\Phi_K\rangle$  is localized in angle and, by the uncertainty principle, is not a state of good angular momentum ( $J$ ). Thus  $|\Phi_K\rangle$  does not have a unique  $J$  quantum number and is a superposition of various  $J$  states [4–7]. One needs to project out states of good angular momenta from the intrinsic state

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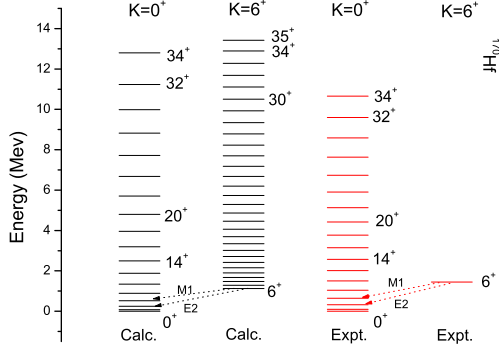


FIG. 2: Energy spectra of  $^{170}\text{Hf}$ . The Experimental observed values are taken from Ref nndc.bnl.gov.

$\Phi_K$  with the Angular Momentum Projection operator,

$$P_K^{IM} = \frac{2I+1}{8\pi^2} \int d\Omega D_{MK}^I(\Omega) R(\Omega). \quad (1)$$

Here  $R(\Omega)$  is the rotation operator. Reduced matrix elements of tensor operator  $T^L$  between projected states  $\psi_{K_1}^{J_1}$  and  $\psi_{K_2}^{J_2}$  are given by

$$\begin{aligned} \langle \psi_{K_1}^{J_1} || T^L || \psi_{K_2}^{J_2} \rangle &= \frac{(J_2 + 1/2)(2J_1 + 1)^{1/2}}{(N_{K_1 K_1}^{J_1} N_{K_2 K_2}^{J_2})^{1/2}} \\ &\sum_{\mu\nu} C_{\mu\nu K_1}^{J_2 L J_1} \times \int_0^\pi d\beta \sin(\beta) d_{\mu K_2}^{J_2}(\beta) \\ &\times \langle \phi_{K_1} | T_\nu^L e^{-i\beta J_y} | \phi_{K_2} \rangle \end{aligned} \quad (2)$$

where the tensor operator  $T^L$  denotes electromagnetic operators of multipolarity  $L$ .

We remark that the reduced matrix element has J-selection rule in the form of Clebsch-Gordan coefficient; but there is no K-selection rule.

## Results and Discussion

The deformed HF orbits are calculated with a spherical core of  $^{132}\text{Sn}$ , the model spans the  $2s_{1/2}$ ,  $1d_{3/2}$ ,  $1d_{5/2}$ ,  $0g_{7/2}$ ,  $0h_{9/2}$  and  $0h_{11/2}$  orbits for protons and the  $2p_{1/2}$ ,  $2p_{3/2}$ ,  $1f_{5/2}$ ,  $1f_{7/2}$ ,  $0h_{9/2}$  and  $0i_{13/2}$  orbits for neutrons respectively.

We use surface delta interaction [8] among the active nucleons to obtain deformed single particle orbits. The calculated results are presented in Fig. 1.

## Conclusions

We have used the deformed HF and angular momentum projection technique to obtain energy spectra of ground and excited  $K=6^+$  bands of  $^{170}\text{Hf}$ . The energy spectra after angular momentum projection are shown in Fig.2. The agreement between the results of our calculations and the experimental spectra of the various bands are reasonable up to moderate spin values. E2 and M1 transitions from the  $K=6^+$  bandhead to the ground band are also shown. In our calculation we find that the E2 and M1 from  $K=6^+$  band to  $K=0^+$  band are finite but retarded by about a factor of  $10^{*-10}$  compared to transitions within the bands, and with no K-mixing. Thus Peierls-Yoccoz procedure has J selection rule; but there is no K-selection rule in this formalism.

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