Semiclassical description of α radioactivity in spherical nuclei

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Introduction

The level density can be written as a sum of average and oscillating part, $g(E) = \tilde{g}(E) + \delta g(E)$. As a result of the connection between oscillating part of level density and the classical periodic orbits, shell effects appear in the level density. Utilizing the level density for spherical harmonic oscillator along with spin-orbit interactions [1], the present work highlights the description of α radioactivity in spherical nuclei such as Sm^{146}, Gd^{148}. Q_α values for these are calculated by introducing shell and pairing corrections to the liquid drop binding energy.

Methodology

Among all the possible decay modes of heavy nuclei, α decay is the most important, because it provides detailed information on nuclear structure, nuclear interaction as well as in the identification of new elements. In this work, we would like to introduce shell structures δU and pairing correlations δP in the binding energy (function of atomic number Z and mass number A) calculation:

$$B(Z,A) = B_{LDM}(Z,A) + \delta U + \delta P. \quad (1)$$

Here, $B_{LDM}(Z,A)$ [2] is given as:

$$\begin{aligned} B_{LDM}(Z,A) &= a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} \\ &- a_a \frac{(N-Z)^2}{A}. \end{aligned}$$

(1) Introducing the shell corrections
The consideration of shell structures is important to incorporate the details of the nuclear structure in terms of the appropriate shell closures. These can be taken care of through the total and only the smooth part of the level density [1] and can be calculated as:

$$\begin{aligned} \delta U(n,p) &= 2\int_0^{E'^{\,n,p}_F} E g_{n,p}(E) dE \\ &- 2\int_0^{E_F^{n,p}} E\tilde{g}_{n,p}(E) dE \quad (2) \end{aligned}$$

where, E'_F and E_F represents the Fermi energy, with and without shell effects respectively.

(2) Introducing the pairing corrections
The tendency of like nucleons to pair up greatly affects the nuclear structure especially at low energies. Nuclear superfluid transitions are quite well known. The pairing correlation energy can be defined as:
$\delta P_{n,p} = |\tilde{E}_{pair} - \tilde{E}|$ where

$$\begin{aligned} \tilde{E}_{pair}(n,p) &= 2\int_{-\infty}^{r} E\tilde{g}_{n,p}(E) dE - \frac{\Delta^2_{n,p}}{G_{n,p}} \\ &+ \int_r^s E\tilde{g}_{n,p}(E)\left[1 - \frac{E-\lambda'_{n,p}}{E_q^{n,p}}\right] dE \\ \tilde{E}(n,p) = &= 2\int_{-\infty}^{E_F^{n,p}} E\tilde{g}_{n,p}(E) dE. \quad (3) \end{aligned}$$

Here, $r, s = \lambda'_{n,p} \mp \hbar\omega_{n,p}$. $\lambda'_{n,p}$ and $E_F^{n,p}$ are the chemical potentials with and without pairing. The quasiparticle energy is defined as, $E_q = ((E - \lambda'_{n,p})^2 + \Delta^2_{n,p})^{1/2}$. $\Delta_{n,p}$ are the pairing gaps obtained from the four-point formula. We took 80% of the pairing gaps value for our calculations. $G_{n,p}$ is the pairing strength assumed constant in the BCS theory. The spacing between the oscillator levels is:

$$\hbar\omega_{n,p} = \frac{41}{A^{\frac{1}{3}}}\left(1 \pm \frac{N-Z}{A}\right)^{\frac{1}{3}} MeV. \quad (4)$$

In the above expressions, the level density for spherical harmonic oscillator along with spin-orbit interactions is used, whose average part is:

$$\begin{aligned}\tilde{g}(E) &= \frac{E^2}{2\hbar^3\omega^3}\left[1+3\kappa^2\hbar^2\omega^2\right] \\ &- \frac{1}{8\hbar\omega}\left[1+5\kappa^2\hbar^2\omega^2\right] \\ &+ E\kappa^3\hbar\omega + O(\hbar^4\kappa^4)+.... \end{aligned} \quad (5)$$

and the corresponding oscillating part employed is as given in [1]. Adding these corrections to liquid drop model $B_{LDM}(Z,A)$ [2], we get the modified binding energy $B(Z,A)$ as stated in eq.(1).

(3) Evaluation of Q_α value

We have considered Sm^{146}, Gd^{148} nuclei, which are assumed to be spherical in their ground states. Each of which is unstable and undergoes alpha decay to form Nd^{142}, Sm^{144} respectively. Q_α value or the α disintegration energy can be used to probe the nuclear structure, as it depends on the nuclear transition from the parent to the daughter state. It can be calculated as:

$$Q_\alpha = B_D(Z,A) + B_\alpha - B_P(Z,A). \quad (6)$$

Here, the binding energies of parent and daughter are evaluated as in eq.(1), while the binding energy of α particle is taken from [3]. The various structure parameters used for the calculations are shown in table 1.

Nucleus	κ_p $(\hbar\omega_p)^{-1}$	κ_n $(\hbar\omega_n)^{-1}$	Δ_p (MeV)	Δ_n (MeV)	(k_p, k_n)
Sm^{146}	-0.065	-0.062	1.08	0.75	(5,9)
Nd^{142}	-0.065	-0.060	0.98	1.10	(5,5)
Gd^{148}	-0.065	-0.062	1.14	0.74	(5,5)
Sm^{144}	-0.065	-0.060	0.99	1.14	(5,5)

Table 1: Spin-orbit strength parameters $\kappa_{p,n}$, pairing gaps $\Delta_{p,n}$ and repetitions over periodic orbits (k_p, k_n) used for our calculations.

Results and Discussion

The obtained shell, pairing corrections, binding energies and the Q_α values are listed in table 2 and 3 respectively.

Nucleus	δU (MeV)	δP (MeV)	$B(Z,A)$ (MeV)	$B_E(Z,A)$ (MeV)
Sm^{146}	-1.9576	2.0060	1209.27	1210.90
Nd^{142}	-0.4694	2.4744	1183.46	1185.14
Gd^{148}	-0.8275	2.1704	1220.59	1220.75
Sm^{144}	-0.1245	2.6265	1195.08	1195.73

Table 2: The shell and pairing corrections δU and δP, obtained $B(Z,A)$ and the experimental values $B_E(Z,A)$ [4].

Parent nucleus	Daughter nucleus	Q_α (MeV)	$Q_\alpha^{Expt.}$ (MeV)
Sm^{146}	Nd^{142}	2.486	2.529
Gd^{148}	Sm^{144}	2.786	3.271

Table 3: The comparison of obtained Q_α and the corresponding experimental value $Q_\alpha^{Expt.}$ [4].

It is observed that the calculated Q_α value matches with the experimental value $Q_\alpha^{Expt.}$ quite well. This Q_α value can yield important information about nuclear structure and it's stability.

References

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