Error analysis of statistical model for fission

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Predictive power of a model is tested by experimental measurements, which testify its credibility to be used as a benchmark. However, phenomenological models with a few parameters employed to reproduce a set of experimental data are not free of errors. It is often the case that an objective function χ^2 is minimized to optimize the set of parameters, which is defined as

$$\chi^2(\mathbf{p}) = \frac{1}{N_d - N_p} \sum_{i=1}^{N_d} \left(\frac{\mathcal{O}_i^{exp} - \mathcal{O}_i^{th}(\mathbf{p})}{\Delta \mathcal{O}_i} \right)^2, \quad (1)$$

where, N_d and N_p are the number of experimental data points and the number of fitted parameters, respectively. $\mathcal{O}_i^{exp}$ and $\mathcal{O}_i^{th}(\mathbf{p})$ are the experimental and the corresponding theoretical values for a given observable. $\Delta\mathcal{O}_i$ is the adopted error which is given by

$$\Delta\mathcal{O}_i = \Delta\mathcal{O}_i^{th} + \Delta\mathcal{O}_i^{exp} + \Delta\mathcal{O}_i^{num}. \quad (2)$$

The terms in the right hand side are theoretical, experimental and numerical errors respectively. Once the minimum value of the χ^2 $(= \chi_0^2)$ corresponding to the optimized parameter set $\mathbf{p}$ $(= \mathbf{p_0})$ is obtained, one can calculate errors on different parmaters as well as observables by employing covariance analysis [1]. We have employed this method of covariance analysis to examine the merits of statistical model for fission.

In its primitive form, a standard statistical model (SM) could reproduce the experimental observables like evaporation residue ($\sigma_{\rm ER}$), fission cross sections ($\sigma_{\rm fiss}$), neutron multiplicity ($\nu_{\rm pre}$) data by tuning its parameters (*viz.* level density parameters at ground state and saddle, a scaling factor for the fission barrier, a pre-saddle delay and saddle-to-scission transition time) on an *ad-hoc* basis [2, 3]. But, eventually after incorporation of parameters like shell correction (in level density and fission barrier), orientation degree of freedom ($K_{\rm or}$), collective enhancement in level density ($K_{\rm coll}$) and a suitable dissipation, the $\sigma_{\rm ER}$, $\sigma_{\rm fiss}$ and $\nu_{\rm pre}$ are simultaneously reproduced for asymmetric reactions populating compound nucleus (CN) of mass $A_{\rm CN}$ up to ~ 200 [4], which had been hitherto uncomprehended. The pre-saddle dissipation strength (β) was the only free parameter in that analysis. There were several other parameters (e.g. parameters that describe the damping of shell correction ($E_{\rm D}$) and collective modes ($E_{\rm cr}$)) which were taken from independent studies assuming that those values would be same in the CN mass region ($A_{\rm CN} \sim 170 - 224$) and different excitation energies (of CN).

So, it was of paramount importance to do an independent error analysis treating β, $E_{\rm D}$, $E_{\rm cr}$ and ΔE (width parameter of the Fermi function defining the collective mode) as free parameters and check whether the values kept fixed, would remain similar in the CN mass $A_{\rm CN} \sim 200$ region at all and they correspond to a global minima or not and to put error bars to the predicted values of the observables.

In the present analysis, the model-I does not take the effects of $K_{\rm or}$ and $K_{\rm coll}$ into account whereas model-II does. So, while only β and $E_{\rm D}$ are varied in case of model-I, β, $E_{\rm D}$, $E_{\rm cr}$ and ΔE are treated as free parame-

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TABLE I: Observables $\mathcal{O}$ of different nuclei, adopted errors on them ($\Delta\mathcal{O}_i$), their experimental values ($\mathcal{O}_i^{exp}$) and the ones ($\mathcal{O}_i^{th}$) obtained for model-I and II.

Reactions	CN	E^*	$\mathcal{O}$	$\mathcal{O}_i^{exp}$	Ref.	$\Delta\mathcal{O}_i$	$\mathcal{O}_i^{th}$ model-I	$\mathcal{O}_i^{th}$ model-II
		107.0	$\sigma_{\rm ER}$	1260±200	[5]	200.0	1316.50±11.30	1324.87±7.49
^{16}O+^{154}Sm	^{170}Yb	107.0	$\sigma_{\rm fiss}$	40±4	[5]	15.0	38.32±9.52	21.32±2.91
		120.8	$\nu_{\rm pre}$	4.4±0.15	[6]	1.0	3.66±0.28	4.40±0.66
	^{192}Pt	72.97	$\sigma_{\rm ER}$	927±129	[7]	129.0	1034.14±2.73	1036.87±3.54
^{16}O+^{176}Yb		99.98	$\nu_{\rm pre}$	4.4±0.5	[8]	1.0	3.37±0.15	4.35±0.40
		72.41	$\sigma_{\rm ER}$	557±21	[9]	200.0	812.53±30.99	611.46±67.89
^{16}O+^{184}W	^{200}Pb	72.41	$\sigma_{\rm fiss}$	398±6	[9]	100.0	190.01±28.69	394.50±72.84
		195.8	$\nu_{\rm pre}$	7.7±0.3	[10]	2.0	7.16±0.84	9.16±0.69

TABLE II: The optimised parameters in different models and their correlated errors. χ^2 per degree of freedom is also mentioned.

Models (χ^2)	β ($\times 10^{21} s^{-1}$)	$E_{\rm D}$ (MeV)	$E_{\rm cr}$ (MeV)	ΔE (MeV)
I (1.19)	1.21±0.40	42.62±1.55	-	-
II (1.30)	1.98±0.43	51.08±36.59	80.10±19.00	10.37±6.95
others	2-4 [11–13]	18.5 [14] 28.57 [15]	40-60 [16, 17]	10 [16]

ters in model-II, to investigate the correlations among them and their influences on the observables. The different experimental observables and adopted errors on them along with their theoretically obtained values for the reactions (^{16}O+^{154}Sm, ^{16}O+^{176}Yb, ^{16}O+^{184}W) populating CNs (^{170}Yb, ^{192}Pt, ^{200}Pb) used for analysis are mentioned in Table I. The optimized parameters, their associated errors, the χ^2 values and their optimized values reported elsewhere are mentioned in Table II for comparison.

The damping of collective enhancement ($E_{\rm cr}$) with excitation energy, being comparable to the damping of the shell effects *i.e.* $E_{\rm D}$, has definitely an influence on the production cross section [16]. Apparently, from Table I and II, the optimized parameters are found to be within their reasonable limits. Model-II gives a better agreement with the measured data. A detailed covariance analysis with more data points, more SM parameters and more events, is underway.

One of the authors (T.B.) acknowledges the support from the theory divisions of VECC and SINP, Kolkata during this work.

References

[1] J. Dobaczewski *et al.*, J. Phys. G: Nucl. Part. Phys. **41**, 074001 (2014).
[2] R. N. Sagaidak and A. N. Andreyev, Phys. Rev. C **79**, 054613 (2009).
[3] Tathagata Banerjee *et al.*, Phys. Rev. C **94**, 044607 (2016).
[4] Tathagata Banerjee *et al.*, Phys. Lett. B **776**, 163 (2018).
[5] A. M. Zebelman and J. M. Miller, Phys. Rev. Lett. **30**, 27 (1973).
[6] D. J. Hinde *et al.*, Nucl. Phys. **A452**, 550 (1986).
[7] R. P. Schmitt *et al.*, Phys. of Atom. Nucl. **66**, 1163 (2003).
[8] Tapan Rajbongshi *et al.*, Phys. Rev. C **93**, 54622 (2016).
[9] C. E. Bemis Jr. *et al.*, ORNL physics division progress report 6326 (1986).
[10] D. J. Hinde *et al.*, Phys. Rev. C **45**, 1229 (1992).
[11] P. Fröbrich *et al.*, Phys. Rep. **292**, 131 (1998).
[12] I. Diószegi *et al.*, Phys. Rev. C **61**, 024613 (2000).
[13] J. P. Lestone *et al.*, Phys. Rev. C **79**, 044611 (2009).
[14] W. Reisdorf, Z. Phys. A - Atoms and Nuclei **300**, 227 (1981).
[15] R. J. Charity *et al.*, Phys. Rev. C **71**, 024310 (2005).
[16] A. R. Junghans *et al.*, Nucl. Phys. A **629**, 635 (1998).
[17] S. Komarov *et al.*, Phys. Rev. C **75**, 064611 (2007).