Relativistic Thomas-Fermi EoS of magnetic White Dwarfs

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Introduction

The Feynman-Metropolis-Teller treatment of compressed atom [1] is extended to the relativistic regime. Each atomic configuration is confined by a Wigner-Seitz cell and is characterized by a positive electron Fermi energy. The nonrelativistic treatment assumes a pointlike nucleus and infinite values of the electron Fermi energy can be attained. In the relativistic treatment, there exists a limiting configuration, reached when the Wigner-Seitz cell radius equals the radius of the nucleus with a maximum value of the electron Fermi energy. These approximations correctly reproduce Chandrasekhar mass limit for White Dwarfs (WDs). But recently several WDs have been proposed with masses significantly greater than this limit, known as Super-Chandrasekhar WDs, to account for the over-luminous Type Ia supernovae [2]. FMT treatment with Coulomb screening in presence of strong quantizing magnetic field has been applied in this work to develop the Equation of State(EoS). The Mass-Radius relations for magnetized WDs are obtained by solving the Tolman-Oppenheimer-Volkoff equations.

EoS for magnetic White Dwarfs

We consider a compressed atom as a Wigner-Seitz cell consisted of a finite sized nucleus at the center of the cell and completely degenerate relativistic electron gas embedded in a strong magnetic field. We consider here the interaction between the nucleus and the electrons. Electrons, being charged particles, occupy Landau quantized states in a magnetic field. Electrons with spin s and charge $q=-|e|$, the maximum number of particles per Landau level per unit area is $\frac{|e|B(2s+1)}{hc}$ in magnetic field B. On solving Dirac's with spin in an external magnetic field B in z-direction which is uniform and static, energy eigenvalues are given by

$$E_{\nu,p_z} = \left[p_z^2c^2 + m_e^2c^4\left(1+2\nu B_D\right)\right]^{\frac{1}{2}} - m_ec^2 - eV(r) \tag{1}$$

where ν=$n+\frac{1}{2}+s_z$, the Landau quantum number, m_e is electron rest mass and the dimensionless magnetic field defined as B_D=B/B_c is introduced with B_c given by $\hbar\omega_c$=$\hbar\frac{|e|B_c}{m_ec}$=m_ec^2 $\Rightarrow$ B_c=$\frac{m_e^2c^3}{|e|\hbar}$=$4.414\times10^{13}$ gauss. A constant distribution of protons confined in a radius given by R_c=$r_0A^{\frac{1}{3}}$ with r_0=$1.2fm$ is assumed. Using Landau quantization, electronic number density is given by

$$\begin{aligned} n_e &= \frac{2B_D}{(2\pi)^2\lambda_e^3}\sum_{\nu=0}^{\nu_m} g_\nu \frac{p_zc}{m_ec^2} \\ p_zc &= \left[\widehat{V}^2\left(1-\frac{\nu}{\nu_m}\right) + 2m_ec^2\widehat{V}\left(1-\frac{\nu}{\nu_m}\right)\right]^{\frac{1}{2}} \end{aligned} \tag{2}$$

where $\widehat{V}=eV+E_{\nu,p_z}$ and Eq.(1) is used for its evaluation. ν_m is the upper limit of Landau level can be found from the condition $p_z^2\geq0$ and is given by $\nu_m=\frac{\widehat{V}^2+2\widehat{V}m_ec^2}{2B_Dm_e^2c^4}$. The overall Coulomb potential outside the nucleus satisfies the poisson equation

$$\begin{aligned} \nabla^2V(r) &= 4\pi e n_e(r) \\ \Rightarrow \frac{1}{x}\frac{d^2\chi(x)}{dx^2} &= \frac{8\pi e^2B_D}{4\pi^2c\hbar}\frac{m_\pi}{m_e}\left(\frac{\lambda_\pi}{\lambda_e}\right)^3 \\ &\sum_{\nu=0}^{\nu_m} g_\nu\left(1-\frac{\nu}{\nu_m}\right)^{\frac{1}{2}}\left[\left(\frac{\chi(x)}{x}\right)^2 + \frac{\chi(x)}{x}\frac{2m_e}{m_\pi}\right]^{\frac{1}{2}} \end{aligned} \tag{3}$$

where dimensionless quantities x=$\frac{r}{\lambda_\pi}$, $\frac{\chi}{r}$=$\frac{\widehat{V}(r)}{c\hbar}$ have been introduced. Solving Eq.(3) we find the electrostatic potential and the electronic

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distribution. Hence the potential energy density ε_p, kinetic energy density ε_k are found. The energy density ε and pressure P expressions are given by,

$$\begin{aligned}\varepsilon_p &= -en_e(r)V(r)\\ \varepsilon_k &= \frac{2B_D m_e c^2}{4\pi^2\lambda_e^3}\sum_{\nu=0}^{\nu_m} g_\nu\left(1+2\nu B_D\right)\\ &\quad \psi\left(\frac{x_F(\nu)}{(1+2\nu B_D)^{1/2}}\right)\\ \varepsilon &= \varepsilon_p+\varepsilon_k+\rho m_B c^2+\frac{B^2}{8\pi} \end{aligned} \quad (4)$$

$$\begin{aligned}P &= P_B+P_e\\ &= \frac{B^2}{24\pi}+\frac{2B_D m_e c^2}{4\pi^2\lambda_e^3}\\ &\quad \sum_{\nu=0}^{\nu_m} g_\nu\left(1+2\nu B_D\right)\eta\left(\frac{x_F(\nu)}{(1+2\nu B_D)^{1/2}}\right)\end{aligned} \quad (5)$$

where $x_F(\nu)$=$p_z/m_e c$, λ_e=$\frac{\hbar}{m_e c}$, λ_π=$\frac{\hbar}{m_\pi c}$, m_π is the pion mass, ρ is the baryonic number density, m_B is the baryonic mass. The magnetic energy contribution is ε_B=$\frac{B^2}{8\pi}$ while P_B=$\frac{\varepsilon_B}{3}$ is the magnetic contribution to pressure and

$$\begin{aligned}\psi(z) &= \int_0^z \left(1+y^2\right)^{\frac{1}{2}}\\ &= \frac{1}{2}\left[z\sqrt{1+z^2}+ln\left(z+\sqrt{1+z^2}\right)\right]\\ \eta(z) &= z\sqrt{1+z^2}-\psi(z)\end{aligned} \quad (6)$$

Calculations and Results

We perform calculations with varying magnetic field inside WD given by the form [3]

$$B_d = B_s + B_0[1-\exp\{-\beta(\rho/\rho_0)^\gamma\}] \quad (7)$$

where B_d (in units of B_c) is the magnetic field at baryonic density ρ, B_s (in units of B_c) is the surface magnetic field and ρ_0 is taken as ρ(r=0)/10 and β, γ are constants. We choose constants β=0.8, γ=0.9, rather arbitrarily but the central and surface magnetic fields once fixed the variations of its profile do not alter the gross results. The maximum central magnetic field strength is kept at $10B_c$ which is 4.414×10^{14} gauss [4] and surface magnetic field at $\sim 10^9$ gauss estimated by observations.

TABLE I: Variations of masses and radii of magnetized WD. The maximum magnetic field B_{dc} at the centre is listed in units of B_c.

ρ (r=0)	Radius	Mass	B_{dc}
fm^{-3}	Kms	$M_\odot$	B_c
9.348×10^{-6}	1285.91	1.4146	1.5
9.348×10^{-6}	1344.46	1.4236	1.75
9.348×10^{-6}	1349.45	1.4339	2.0
9.350×10^{-6}	1388.04	1.4906	3.0
9.344×10^{-6}	1438.94	1.5731	4.0
9.342×10^{-6}	1503.64	1.6863	5.0
9.355×10^{-6}	1581.27	1.8353	6.0
9.355×10^{-6}	1663.86	2.0217	7.0
9.331×10^{-6}	1758.40	2.2601	8.0
9.323×10^{-6}	1954.44	2.8997	10.

TABLE II: Variations of masses and radii of magnetized WD with coulomb interaction. The maximum magnetic field B_{dc} at the centre is listed in units of B_c.

ρ (r=0)	Radius	Mass	B_{dc}
fm^{-3}	Kms	$M_\odot$	B_c
6.039×10^{-7}	3572.24	2.2653	1.5
6.039×10^{-7}	3142.19	1.8405	1.2
6.039×10^{-7}	2801.06	1.5725	0.9
6.039×10^{-6}	2110.46	3.2624	8.0
6.039×10^{-6}	1659.09	2.0178	5.0
6.039×10^{-6}	1369.83	1.5157	2.0

Summary and Conclusion

The EoS for magnetized WD in presence of Coulomb screening has been explored as further refinement [5]. We find that the inclusion of Coulomb interaction modifies the masses of WD further upward and significantly greater than Chandrasekhar limit.

References

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