Limiting mass of a neutron star

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Introduction

Neutron stars and white dwarfs are strongly degenerate compact matters. If the mass of the dead core of a white dwarf exceeds Chandrasekhar limit, the electron degeneracy pressure of the white dwarf can not balance its gravitational pressure, leading to further inward collapse of the core. This causes an increase in the temperature of the core and the corrsponding Fermi energy of the degenerate electrons exceeds the energy difference between neutron and proton, promoting inverse beta decay in the core. In this nuclear process proton is converted into neutron and neutrino ($p+e^-=n+\nu_e$). The core then ends up being a neutron star. The limiting maximum mass for a neutron star (for a white dwarf as well) has been highly debated in the literature. Oppenheimer and Volkov calculated the limiting mass of a neutron star to be 0.7 times of solar mass [1]. This result turned out to be very low compared to the Chandrasekhar limit since the authors assumed that the neutrons follow a Fermi Dirac distribution for ideal gas. Different stellar observations of pulsars led to an idea that the maximum mass of neutron star may lie between 1.4 to 3.0 solar mass [2]. Integrating the equation of equilibrium Rhoades and Ruffini found that the maximum mass of neutron star to be 3.2 solar mass [3]. Nauenberg and Chapline found the limit to be 3.6 solar mass [4]. Rotating neutron stars may have higher mass limit. [5] In our work we have used stellar equation of hydrostatic equilibrium to find the limiting mass for a non-rotating main sequence star to remain as a neutron star where we have used an empirical relation for the density of the star as a function of the radius of the star.

Theory

Let us begin with the stellar equation of hydrostatic equilibrium for a main sequence star as

$$\frac{dP}{dr}=\frac{-GM(r)\rho(r)}{r^2} \quad (1)$$

We introduce an empirical relation for the density of the star,

$$\rho(r)=\rho_0\left[1-\left(\frac{r}{R}\right)^2\right] \quad (2)$$

This empirical relation satisfies the conditions $\rho(r)=0$ at the surface of the star and $\rho(r)=$maximum ρ_0 at the center. Therefore the mass of the star,

$$M(r)=\int_0^r 4\pi r^2\rho(r)\,dr=\frac{8\pi}{15}\rho_0 r^3 \quad (3)$$

Using equations (2) and (3) it follows

$$\frac{dP}{dr}=\frac{-G\left[4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5R^2}\right]\right]\left[\rho_0\left[1-\left(\frac{r}{R}\right)^2\right]\right]}{r^2} \quad (4)$$

Integrating we have,

$$P_C=\frac{-4\pi G\rho_0^2}{15}R^2 \quad (5)$$

Where, P_C represents the pressure at the center of the core of the star.
The equation (5) can be taken as most probable equation of state of a star.
Now, considering Fermi Dirac distribution for the neutrons present in the star, the neutron degeneracy pressure can be calculated as follows:

$$P_n=\frac{1}{3}\int_0^{p_F} pv\,n(p)\,dp \quad (6)$$

where n is the number density of component (neutron in this case) of the star. The degeneracy pressure in extreme relativistic limit of the velocity is given by,

$$P_n=\frac{1}{3}\int_0^{p_F} p\,c\left(\frac{2}{h^3}\right)4\pi p^2\,dp =\left(\frac{hc}{8}\right)\left(\frac{3}{\pi}\right)^{\frac{1}{3}}\left(\frac{\rho}{m\mu}\right)^{\frac{4}{3}} \quad (7)$$

where μ and m are the average molecular weight and the mass of the component of the star respectively. Equating equation (5) and (7) we can find the extreme value for the mass of a stable neutron star. We then have,

$$\frac{4\pi}{15}\rho_0^2 G R^2 = \left(\frac{hc}{8}\right)\left(\frac{3}{\pi}\right)^{\frac{1}{3}}\left(\frac{\rho_0}{m\mu}\right)^{\frac{4}{3}} \qquad (8)$$

Results and Discussions

Solving equation (8) and substituing r=R in equation (3) we get ,

M=5.47×10^{30} Kg = 2.75 solar mass. (9)

Neutron star with mass greater than this limit may not be able to retain its stability and it will ultimately collapse to a black hole or any other form of compact matter. At M=2.75 solar mass the radius of the star becomes 8.108 Km using the condition of Schwarzschild radius. The relation (8) may also be used to find the limiting mass of white dwarf. In hydrogen depleted white dwarf, the dead core contains primarily helium along with some traces of heavy nuclei and unburnt hydrogen, caused by nuclear fission reactions. Ignoring the unburnt hydrogen, if assume that the remnant core is composed with 100% helium, then μ=0.75 and limiting mass of white dwarf is 1.547 solar mass. If it contains 90% helium and 10% heavy nucleus, then μ=0.725 and the limiting mass becomes 1.445 solar mass. This is in well agreement with the Chandrasekhar limt.

Conclusions

The result from different theoretical and experimental observation the upper mass limit of neutron star lies in a range of 1.4 solar mass to 3.6 solar mass. [5] Using an empirical equation for the density of a main sequence star we have estimated the limiting mass for a neutron star to be 2.75 solar mass which is in very good agreement with the previous theoretical predictions. The equation of state derived here may be used as an equation of state for the stars suffering gravitational collapse after discharging a supernova explosion with less approximation.

References

[1] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev., **55**, 507 (1939).
[2] G.A.Carvalho et al., J.Phys.: Conf.Ser., **630** 012058 (2015).
[3] C. E. Rhoades and R. Ruffini, Phys. Rev. Lett., **32**, 324 (1974).
[4] M. Nauenberg and G. Chapline, Jr., Astrophys. J., **179**, 417 (1973).
[5] The maximum mass of neutron stars, G, Srinivasan, Bull. Astr. Soc. India **30**, 523 (2002).