Algebraic calculations of mass spectra of scalar and axial vector charmonia in framework of Bethe-Salpeter equation

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Abstract: In this work we calculate the mass spectrum of charmonium for 1P,...,4P states of 0^{++} , and 1^{++}states, in the framework of a QCD motivated Bethe-Salpeter Equation. In this 4 x 4 BSE framework, the coupled Salpeter equations are first shown to decouple for the confining part of interaction, under heavy-quark approximation, and analytically solved. Then, the one-gluon-exchange interaction is perturbatively incorporated, leading to mass spectral equations for various quarkonia. The analytic forms of wave functions obtained can be used for calculation of various transition amplitudes, such as two-photon decay widths of χ_{c0}. Our results are in reasonable agreement with data (where ever available) and other models.

1. Introduction: The conventional heavy $Q\bar{Q}$states such as $c\bar{c}$and $b\bar{b}$ are well understood to be quark- anti quark pair bound by the coulombic potential, that arises due to the perturbative one-gluon-exchange that dominates its short range part, and the linearly rising part that describes the confining potential at long distances, and are crucially important to improve our understanding of QCD. There has been a renewed interest in recent years in spectroscopy of these heavy hadrons in charm and beauty sectors, which was primarily due to experimental facilities the world over such as BABAR, Belle, CLEO, DELPHI, BES etc. [1], which have been providing accurate data on $c\bar{c}$and $b\bar{b}$ hadrons with respect to their masses and decays. We do understand that in $Q\bar{Q}$ quarkonia, the constituents are close enough to each other to warrant a more accurate treatment of the OGE (coulomb) term. Though for $b\bar{b}$ systems, the coulomb term will be extremely dominant in comparison to confining term, and should not be treated perturbatively. However for $c\bar{c}$ systems, it will not be unreasonable to treat the coulomb term perturbatively, which can also be seen from our results [2].

2. Calculation: We start with the 4D form of Bethe-Salpeter equation, which using a sequence of steps outlined in [2], are reduced to a set of four Salpeter equations (in 4D variable, q-hat) under Covariant Instantaneous Ansatz (CIA)- which is a Lorentz-invariant generalization of Instantaneous Approximation (IA). These are covariant forms of Salpeter equations which are valid for hadrons in arbitrary motion. Thus, in the present paper, the coupled Salpeter equations for scalar (0^{++}), and axial vector (1^{++}) quarkonia are first shown to decouple for the confining part of interaction, under heavy-quark approximation, and the analytic forms of mass spectral equations are worked out, which are then solved in approximate harmonic oscillator basis to obtain the unperturbed wave functions for various states of these quarkonia. We then incorporate the one-gluon-exchange perturbatively into the unperturbed spectral equation, and obtain the full spectrum. The wave functions of scalar (0^{++}) and axial vector (1^{++}) quarkonia, are also calculated. To solve these equations, we need the BS kernel, which is taken to be one-gluon exchange like as regards the colour and spin dependence, while the potential V($\hat{q},\hat{q}'$) involves the scalar structure of the gluon propagator in the perturbative (o.g.e), as well as the non-perturbative (confinement) regimes. The Salpeter equations lead to a set of coupled integral equations in the amplitudes associated with various Dirac structures. With the form of the confining interaction employed, these coupled integral equations were shown to decouple in the heavy-quark approximation in [2], after which the one-gluon-exchange (OGE) potential was perturbatively incorporated. The resulting mass spectral equations for scalar (0++) and axial vector (1++) quarkonia are [2]

$$E_{S,A}\phi_{S,A}(\hat{q}) = (-\beta_{S,A}^4 \vec{\nabla}_q^4 + \hat{q}^2 + V_{Coul.}^{S,A})\phi_S(\hat{q}), \quad (1)$$

which leads to the complete mass spectral equation,

$$\left(\frac{M^2}{4} - m^2 + \frac{\beta_{S,A}^4 C_0}{\omega_0^2}\right) + 2\beta_{S,A}^2\gamma < V_{coul.}^{S,A} > = 2\beta_{S,A}^2(N + \frac{3}{2}) ;$$

$$N = 2n + l; n = 0,1,2, ..., \quad (2)$$

The analytic forms of the unperturbed wave functions for 1P,…,4P states of scalar (S) and Axial vector (A) quarkonium that are obtained as algebraic solutions of Eqs.(1) (without the coulomb term, $V_{Coul.}^{S,A}$ are given in Eq.(3) below,

$$\phi_{S,A}(1P, \hat{q}) = \left(\frac{2}{3}\right)^{1/2} \frac{\hat{q}}{\pi^{3/4}\beta^{5/2}} e^{-\frac{\hat{q}^2}{2\beta^2}}; \quad (3)$$

$$\phi_{S,A}(2P, \hat{q}) = \left(\frac{5}{3}\right)^{\frac{1}{2}} \frac{\hat{q}}{\pi^{\frac{3}{4}}\beta^{\frac{5}{2}}}(1 - \frac{2}{5}\frac{\hat{q}^2}{\beta^2})e^{-\frac{\hat{q}^2}{2\beta^2}};$$

$$\phi_{S,A}(3P, \hat{q}) = \left(\frac{70}{171}\right)^{\frac{1}{2}} \frac{\hat{q}}{\pi^{\frac{3}{4}}\beta^{\frac{5}{2}}}(1 - \frac{2}{5}\frac{\hat{q}^2}{\beta^2} + \frac{4\hat{q}^4}{35\beta^4})e^{-\frac{\hat{q}^2}{2\beta^2}};$$

$$\phi_{S,A}(4P, \hat{q}) = \left(\frac{1890}{46359}\right)^{\frac{1}{2}} \frac{\hat{q}}{\pi^{\frac{3}{4}}\beta^{\frac{5}{2}}}(1 - \frac{6}{5}\frac{\hat{q}^2}{\beta^2} + \frac{12\hat{q}^4}{35\beta^4} - \frac{24\hat{q}^6}{945\beta^6})e^{-\frac{\hat{q}^2}{2\beta^2}};$$

with the expectation value of the coulomb term between the various unperturbed (nP) states in Eq.(2) being, $< nP|V_{coul.}^{S,A}|nP >= -\frac{128\pi\alpha_s}{9\beta_{S,A}^2}$.

3. Results and Conclusions: It is observed that the mass spectra of mesons of various J^{PC} is somewhat insensitive to a range of variations of parameter $\omega_0 \epsilon$[0.130- 0.160] GeV.,as long as C_0/ω_0^2 is a constant, and reasonably good fits are obtained for $\omega_0 = 0.145\ GeV.$, C_0 =0.15022, and charmed quark mass, $m_c = 1.490\ GeV.$[2]. The mass spectral predictions for 1P,…, 4P states for 0^{++}, and 1^{++} states are given in Tables I and II respectively.

	BSE-CIA	Expt.[1]
$\chi_{c0}(1P)$	3.4056	3.4140± 0.0003
$\chi_{c0}(2P)$	4.1366	
$\chi_{c0}(3P)$	4.8463	
$\chi_{c0}(4P)$	5.5186	

Table I: Mass spectral predictions (in GeV.) of scalar (0^{++}) charmonium for 1P,…,4P states, along with experimental results.

	BSE-CIA	Expt.[1]
$\chi_{c1}(1P)$	3.4783	3.5106±0.00005
$\chi_{c1}(2P)$	3.9560	3.8717±0.00017
$\chi_{c1}(3P)$	4.4650	
$\chi_{c1}(3P)$	4.9593	

Table II: Mass spectral predictions (in GeV.) of axial vector (1^{++}) charmonium for 1P,…,4P states, along with experimental results.

The validity of our approach can be justified by the fact that the plots of our wave functions for states 1P,…, 4P for scalar and axial vector mesons (given in Ref. [2]), obtained analytically in Eq.(3) by decoupling the coupled integral equations and solving the mass spectral equation by working in approximate harmonic oscillator basis is very similar to the corresponding plots of wave functions obtained using a purely numerical approach of solving coupled integral equations. The transparency of our approach (for details see [2-4]) that relies on purely algebraic treatment, with spectrum dependent on principal quantum number, N gives more insight into the mass spectral problem than the purely numerical approaches prevalent in the literature.

4. References:

[1] M.Tanabashi et al., (Particle Data Group), Phys. Rev. D98, 030001 (2018).
[2] S.Bhatnagar, L.Alemu, Phys. Rev. D97, 034021 (2018), and references therein.
[3] H.Negash, S.Bhatnagar, Intl. J. Mod. Phys. E25, 1650059 (2016).
[4] S.Bhatnagar, E.Gebrehanna, K.Shandil in Horizons in World Physics (Ed. A.Reimer, Nova Science Pub. N.Y. USA), Vol. 296, 207-235 (2018).