Mass Spectrum of low lying Bottomonium states in a Non-Relativistic Quark Model with Yukawa Potential

A P Monteiro[1], Praveen P D'Souza [1 2],
Vidyalakshmi N[1],* and K. B. Vijaya Kumar [2]
[1]*P.G Department of Physics, St Philomena College, Darbe, Puttur-574202,INDIA and*
[2]*Deptartment Physics, Mangalore University,*
Mangalagangothri P.O., Mangalore - 574199, INDIA

Introduction

The Yukawa potential, sometimes called the screened Coulomb potential, has a wide range of applications in many branches of physics. Originally, Bottomonium mesons, composed of beauty quarkantiquark pairs bound to each other through the strong force, play a special role in our understanding of hadron formation because the large quark mass allows important simplifications in the relevant theoretical calculations. Yukawa proposed this potential to describe the interaction between a pair of nucleons. However, it is often used as a first approximation of the interaction between two quarks. To better understand the properties of bottomonium, both the progress of experimental measurements and the calculation of theoretical and phenomenological methods are necessary. We also predict the masses of higher bottomonia. Bottomonia system is known to have non-relativistic nature it can be treated as two body system of heavy quark and anti-quark in quantum mechanics. Many experimental groups such as BABAR, Belle, CLEO-III, ATLAS, CMS and HERA-B are producing and expected to produce more precise data [1].

Theoretical Background

The NRQM should work better for heavy quark mesons, since a particle of mass m, localised in a volume of radius R, has a momentum $1/R$ through the uncertainty relation its kinetic energy ($< T >$) $<< m$ only if $mR >> 1$. In the constituent quark models this is satisfied for the c, b and t quarks. Also, in NRQM the spurious excitation of the centre-of-mass (CM) motion can be eliminated easily. Hence, in heavy meson spectroscopy non relativistic models are found to be more suitable in studying the mass spectra. In all potential models the entire dynamics is governed by a Hamiltonian which is composed of a kinetic energy term K and potential energy term V that takes into account the interaction between the quark and the anti-quark. In most of these works potential energy comprises of two important components a short range force and a long range quark confining force.The Hamiltonian employed in our model is given by [3].

$$H_{NRQM} = K + V_{\text{CONF}}(r_{ij}) + V_{\text{YUK}}(r_{ij}), \quad (1)$$

where

$$K = \left[\sum_{i=1}^{2} M_i + \frac{P_i^2}{2M_i}\right] - K_{\text{cm}} \quad (2)$$

where M_i and P_i are the mass and momentum of the ith quark respectively. The K is the sum of the kinetic energies including the rest mass minus the kinetic energy of the centre of mass motion (CM) of the total system. The potential energy part consists of confinement term V_{CONF} In most of the cases quark-antiquark potentials ($V(r)$) are reasonably considered inorder to correlate results with experiment. Quark-antiquark potential may be of the form; The Yukawa potential is the potential that arises from a massive scalar field. The Yukawa potential is given by[4],

$$V_{YUK}(r) = -V_0 \frac{e^{-ar}}{r}$$

and thus can be seen as a screened version of the Coulomb potential, with V_0 describing the strength of the interaction and 1/a its range. Yukawa potential decreases exponentially depending on the value of a.

$$V_{CONF}(\vec{r}_{ij}) = -a_c r_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \quad (3)$$

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*Electronic address: praviphysics@gmail.com

where a_c is the confinement strength and λ_i and λ_j are the generators of the color SU(3) group for the i^{th} and j^{th} quarks. It should be noted that the two body confinement potential, has symmetric and antisymmetric terms. In order to obtain the $Q\bar{Q}$ spectrum, The spin orbit part of the confinement potential is given by

$$V_{so}^{conf} = \sum_{i>j} -\frac{1}{4m^2 r_{ij}} \frac{dV^{conf}}{dr_{ij}} \{(r_{ij}\times p_{ij}) \cdot (\sigma_i + \sigma_j) + [r_{ij} \times (p_i + p_j)/2] \cdot (\sigma_i - \sigma_j)\} \quad (4)$$

If one includes both the symmetric and antisymmetric terms the two terms of V_{ij}^{conf} cancel each other and give an almost vanishing contribution to the single baryon/meson spectra. Also, it should be noted that the symmetric term of the V_{ij}^{conf} has opposite sign to the one in V_{ij}^{OGEP} and hence cancels the contribution from the symmetric term of V_{ij}^{conf} in single baryon/meson system. The antisymmetric term is Galilei non-invariant. It should be noted that the spin orbit term in V_{ij}^{conf} comes from the relativistic effects. The above two body potential is having problems with the long range attractive color Van der Waals force. When it is used for the two hadron system. Since the attraction between color singlet hadrons comes from the virtual excitation of the color octet dipole state of each hadron, this problem is evaded by restricting the model space to describe hadrons.

We have solved the Shrodinger equation with Hamiltonian equation using the variational method:

$$E(\psi) = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} = \langle H\rangle$$

Results and Discussions

In this paper we have calculated the masses of S wave states of Bottomonia considering Yukawa plus Linear confining potential. In our work, we have used the three-dimensional harmonic oscillator wave function which has been extensively used in atomic and nuclear physics is used as the trial wave function for obtaining the $Q\bar{Q}$ mass spectrum.

$$\psi_{nlm}(r,\theta,\phi) = N\left(\frac{r}{b}\right)^l L_n^{l+1/2}(\frac{r}{b}) exp\left(-\frac{r^2}{2b^2}\right) Y_{lm}(\theta,\phi)$$

where $|N|$ is the normalizing constant given by

$$|N|^2 = \frac{2\alpha^3 n!}{\sqrt{\pi}} \frac{2^{(2(n+l)+1)}}{(2n+2l+1)!}(n+l)! \quad (5)$$

Table 1 The Mass Spectrum of Charmonim (MeV)

$n^{2S+1}L_J$	The Mass	M_{exp} MeV	[2] MeV
1 $^1S_0$	9468.96	9398.0±3.2	9402
2 $^1S_0$	9960.99	9999.0±3.5	9976
3 $^1S_0$	10301.1	....	10336
4 $^1S_0$	10594.7	....	10623
5 $^1S_0$	10860.6	....	10869
6 $^1S_0$	11107	....	11097
1 $^3S_1$	9394.28	9460.30±0.26.	9465
2 $^3S_1$	10005	10023.26±0.31	.10003
3 $^3S_1$	10607.2	10355.2±0.5.	10354
4 $^3S_1$	10699.9	10579.4±1.2	10635
5 $^3S_1$	10770.5	10876±11	10878
6 $^3S_1$	11278	11019±8	11102

and $L_n^{l+1/2}(x)$ are the associated Laguerre polynomials,It can be seen that the mass spectra calculated is in close resemblance with the latest PDG results and that with the results of other available theoretical approaches, thus it can be concluded that this potential gives satisfying results for S wave spectra using variational approach.There are theoretical uncertainty in the mass of the heavy quark, α_s and wavefunction at the origin.

References

[1] Ajay Kumar Rai and Raghav Chaturvedi Journal of Physics: Conf. Series 934 (2017)

[2] JS. Godfrey, K. Moats, Bottomonium mesons and strategies for their observation, Phys. Rev. D 92 (2015)

[3] Bhaghyesh and K. B. Vijjyaya Kumar, International Journal of Modern Physics A 27 (2012) 1250127.

[4] Antony Prakash Monteiro, Manjunath Bhat and K. B. Vijaya Kumar. Physics Review D, 054016 **95** (2017)