The study of HPGe detector efficiency using Unscented Transformation Method

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Introduction

Unscented Transformation (UT) method was first introduced by Julier and Uhlmann [1] and was originally developed to improve estimates provided by the extended Kalman filter. It also have been used extensively in control engineering for error propagation by predicting means and covariances in non-linear systems. UT method is generally based on the assumption that approximating a probability distribution is lot more easier than to approximate an arbitrary nonlinear function [2]. A set of samples are chosen deterministically such that their mean and covariance match the probability distribution (not necessarily Gaussian distribution) of input variables. In this paper the uncertainty propagation using UT method has also been done.

A. Uncertainty propagation using Unscented Transform (UT) method:

Unscented transform method is a deterministic method which is easy to implement and does not require analytical linearization steps. In this method a set of points (called sigma points) has been generated that approximates the *pdf* of input parameters. These sigma points are transformed through the nonlinear function resulting in a set of transformed sigma points whom means and covariances are calculated. UT method sounds similar to the MC method, but has small set of vectors.

Consider a primary variable vector $\mathbf{x}$ with mean vector $\bar{x}$ and covariance matrix P. In UT method, we find a set of deterministic vectors, called sigma points, whose ensemble mean and covariance are same as that of $\mathbf{x}$. The known nonlinear functional relationship $\mathbf{y} = \mathbf{f}(\mathbf{x})$ can be used to obtain m dimensional transformed vectors $\mathbf{y}$. For $n \times 1$ vector $\mathbf{x}$, $2n$ sigma points $x^{(i)}$ can be obtained as

$$x^{(i)} = \bar{x} + \widetilde{x}^{(i)}, \quad i = 1, 2, \ldots, 2n \tag{1}$$

where

$$\widetilde{x}^{(i)} = \left(\sqrt{nP}\right)_i, \; \widetilde{x}^{(n+i)} = -\left(\sqrt{nP}\right)_i, \; i = 1, \ldots, n, \tag{2}$$

and $\left(\sqrt{nP}\right)_i$ is the i^{th} column vector of $\sqrt{nP}$. Square root of the matrix nP can be calculated using Cholesky factorization. Transformed vectors, $y^{(i)}$, are calculated using these sigma points by applying functional relationship $\mathbf{y} = \mathbf{f}(\mathbf{x})$. The mean vector and covariance matrix are given as

$$\bar{\mathbf{y}} = \sum_{i=1}^{2n} W^{(i)} y^{(i)} \tag{3}$$

and

$$\mathbf{Cov} = \sum_{i=1}^{2n} W^{(i)} (y^{(i)} - \bar{\mathbf{y}})(y^{(i)} - \bar{\mathbf{y}})^T, \tag{4}$$

where $W^{(i)}$ are weighting coefficients defined as $W^{(i)} = \frac{1}{2n}$. According to Savin et. al [3], UT method does not require the Jacobian matrix and better suited than linearization. It can be fine tuned to higher order moments of the approximation to reduce the overall prediction error. In case of Gaussian variables, first two moments sufficiently approximates the *pdf* of the required variable [3, 4].

Efficiency calibration of HPGe γ-ray detector:

The energy-efficiency calibration of HPGe detector was carried out using a standard

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TABLE I: :Mean values (ϵ) and standard deviation ($\delta\epsilon$) of HPGe detector's efficiency and its corresponding covariance matrix using UT method

Efficiency		Covariance matrix ($\times E-10$)								
ϵ	$\delta\epsilon$									
0.121	0.124E-03	0.903								
0.245	8.502E-03	0.241	0.524							
0.344	6.384E-03	0.181	0.124	0.324						
0.779	3.075E-03	8.734	5.981	4.491	9.710					
0.964	2.654E-03	7.537	5.162	3.876	1.867	5.952				
1.086	2.377E-03	6.752	4.624	3.472	1.673	1.443	3.954			
1.112	2.340E-03	6.646	4.551	3.417	1.646	1.421	1.273	4.040		
1.299	2.028E-03	5.762	3.946	2.963	1.428	1.232	1.104	1.0865	1.780	
1.408	1.913E-03	5.435	3.722	2.795	1.346	1.162	1.041	1.025	8.881	2.570

point γ-ray source of ^{152}Eu. The efficiency of the detector was measured by six gamma lines of ^{152}Eu used as calibration source. The measured efficiency corresponding to each of the six gamma lines was given by the equation

$$\varepsilon = \frac{C}{A_o I_\gamma e^{\left(-\frac{0.693}{T_{1/2}}t\right)}} \tag{5}$$

where

ε is the efficiency of detector,

C is the γ-ray counts,

I_γ is the branching factor or γ-ray abundance,

A_o is activity at the time of source calibration,

t is the time elapsed between source calibration and detector calibration,

$T_{1/2}$ is the half life of Eu. The experimental values for this calculation was taken from Imran et al., [5]

Conclusion

In this paper, we have done the calculation of mean values of HPGe detector's efficiency using UT method. The requisite numerical data for calculations are obtained from [5]. The efficiency of the detector along with the standard deviation and covariance matrix are given in the Table I. The values are almost same as that given by the Sandwich formula. The UT method can serve as a great alternative to the Sandwich formula for propagation of uncertainties whenever it does not require Jacobian matrix and better suited than linearization. It will be of great interest to further examine the applications and performance of the UT method in complex reactor physics calculations.

References

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