Semiclassical aspects of cluster decay in ^{252}Cf

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Introduction

Cluster radioactivity is known to be the fourth type of radioactivity, defined as the spontaneous decay in which fragments heavier than α particle are emitted. The phenomenon of cluster decay was first theoretically predicted by Sandulescu et al. utilizing Quantum mechanical fragmentation theory(QMFT) and later it was experimentally confirmed by Rose and Jones in 1984 [1,2]. Since then there has been rapid advancements both theoretically and experimentally to study light as well as heavier cluster emission from radioactive nuclei [3].

In this work, the aspects of cluster decay for various clusters such as ^4He, ^{42}S, ^{46}Ar which may be emitted in the cluster decay of ^{252}Cf is discussed. For this purpose we have calculated the disintegration energy(Q-value) within the microscopic-macroscopic approach [4]. The shell corrections to the binding energy used as input in the Q-values are evaluated by utilizing the semiclassical trace formula for spherically symmetric harmonic oscillator potential with spin-orbit interactions [5,6].

Description of cluster radioactivity in ^{252}Cf

We know, like alpha decay, cluster decay can be considered as a tunnelling through an interaction potential barrier $V(r)$. The calculation of disintegration energy $Q-$value for cluster decay is carried out using the recently proposed microscopic-macroscopic approach [4], with the inclusion of shell corrections through the semiclassical techniques [5,6] as:

$$Q = B_1(Z_1, A_1) + B_2(Z_2, A_2) - B(Z, A) \quad (1)$$

where, the calculations of binding energies of parent $B(Z, A)$, daughter and cluster fragments $B_1(Z_1, A_1), B_2(Z_2, A_2)$ is carried out as described in [4]:

$$\begin{aligned} B'(Z,A) &= a_v(1-k_vI^2)A - a_s\left(1-k_sI^2\right) \\ \left(1+\frac{2}{5}\beta_2^2\right)A^{\frac{2}{3}} &- a_c\frac{e^2Z^2}{r_0A^{\frac{1}{3}}}\left(1-\frac{1}{5}\beta_2^2\right) + a_d\frac{Z^2}{A} \\ &- a_w|N-Z|\exp^{-(A/50)^2} \\ &+ a_{w'}\exp^{-80I^2}, \quad (2) \\ B(Z,A) &= B^{'}(Z,A) + \delta U. \quad (3) \end{aligned}$$

where $I = \frac{N-Z}{A}$ is the isospin factor, β_2 is the quadrupole deformation parameter [8], $r_0 = 1.2271$ fm and $e^2 = 1.44$ MeV fm. The first three terms include volume, asymmetry, surface and coloumb energy with $a_v = 15.3982$ MeV, $k_v = 1.7546$, $a_s = 17.3401$ MeV, $k_s = 1.5981$ and $a_c = 0.6$. While the fourth term represents the diffuseness correction to the sharp radius coulomb energy having $a_d = 1.0867$ MeV. Wigner's supermultiplet theory based on SU(4) spin-isospin symmetry was utilized to introduce fifth term with, $a_w = 0.356$ MeV while the sixth term describes the neutron-proton pairing at zero temperature where $a_{w'} = 1.359$ MeV. The consideration of shell structures δU is important to study cluster decay to incorporate the details of the nuclear structure in terms of the appropriate shell closures. These can be taken care of through the total and only the smooth part of the level density [5,6] and can be calculated as:

$$\begin{aligned} \delta U(n,p) &= 2\int_0^{E_F'^{\,n,p}} Eg_{n,p}(E)dE \\ &- 2\int_0^{E_F^{n,p}} E\tilde{g}_{n,p}(E)dE \quad (4) \end{aligned}$$

where, E_F' and E_F represents the Fermi energy, with and without shell effects respectively, evaluated from the neutron and proton number equations as:

$$
\begin{aligned}
N, Z &= \int_{-\infty}^{E_F'^{n,p}} (\tilde{g}(E) + \delta g(E)) dE \\
N, Z &= \int_{-\infty}^{E_F^{n,p}} \tilde{g}(E) dE, \qquad (5)
\end{aligned}
$$

In the above expressions, the level density for spherical harmonic oscillator along with spin-orbit interactions [6] is used, whose average part is:

$$
\begin{aligned}
\tilde{g}(E) &= \frac{E^2}{2\hbar^3\omega^3}\left[1 + 3\kappa^2\hbar^2\omega^2\right] \\
&- \frac{1}{8\hbar\omega}\left[1 + 5\kappa^2\hbar^2\omega^2\right] \\
&+ E\kappa^3\hbar\omega + O(\hbar^4\kappa^4) + ... \qquad (6)
\end{aligned}
$$

where κ is the spin-orbit interaction strength parameter taken in units of $(\hbar\omega_0)^{-1}$ and the corresponding oscillating part employed is as given in [6]. Instead of using BCS formalism, the pairing energy term is simply approximated by the liquid drop term ± 33.6 $A^{-3/4}$(for even-even and odd-odd nuclei respectively).
The spacing between the oscillator levels is chosen as [7]:

$$
\hbar\omega_0(n, p) = \frac{41}{A^{\frac{1}{3}}}\left(1 \pm \frac{N - Z}{A}\right)^{\frac{1}{3}} \text{MeV}.
$$

Results and Discussion

Parent Nucleus	Emitted Cluster	Q (MeV)	Q_B (MeV)
^{252}Cf	^{4}He	1.36	6.22
	^{42}S	103.98	107.97
	^{46}Ar	120.86	126.71

Table 1: Calculated Q-values, Q_B values from [3], for the various emitted clusters are listed in this table.

The structure parameters κ, k [5,7] and the shell corrections δU employed in the calculations are given in table 2. The Q-values for various possible clusters, emitted during cluster decay of ^{252}Cf are calculated and compared with those of Biju et al. [3] are given in table 1.

Nucleus	κ_p $(\hbar\omega_0(p))^{-1}$	κ_n $(\hbar\omega_0(n))^{-1}$	(k_p, k_n)	δU (MeV)
^{252}Cf	-0.060	-0.062	(5,9)	-4.128
^{248}Cm	-0.060	-0.062	(5,9)	-5.421
$^{4}He(N = Z)$	-0.080		4	5.538
^{210}Pb	-0.060	-0.062	(5,9)	0.221
^{42}S	-0.105	-0.090	(6,8)	-0.246
^{206}Hg	-0.060	-0.060	(5,5)	0.038
^{46}Ar	-0.105	-0.075	(6,5)	-3.256

Table 2: Spin-orbit strength parameters $\kappa_{p,n}$, parameter (k_p, k_n) are given in this table.

It is observed that the calculated values are in good agreement with [3].

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