Bandhead Energies of 3QP Quadruplets in ^{175}Lu and ^{177}Lu

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Introduction

In case of odd-A axially symmetric deformed nucleus, the coupling of three-quasiparticles (3QP) in particular Nilsson states having K values, say, K_1, K_2 and K_3, leads to a quadruplet with resultant $K=|K_1 \pm K_2 \pm K_3|$. These four intrinsic states split up due to the residual neutron-proton (*n*-*p*) interactions among the three nucleons [1]. In order to estimate the bandhead energies of the above said four intrinsic 3QP states, Jain and Jain proposed an empirical model [1] but in this model diagonal contributions of particle-particle coupling (ppc), rotor particle coupling (rpc) and irrotational contributions appearing from the interactions of valence particles were ignored. In this paper, we revisited the earlier model [1] by including all the above said contributions and present the band head energy calculations of 3QP quadruplets corresponding to four 3QP configurations observed in ^{175}Lu and ^{177}Lu nuclides.

The Model

The present model is an extension of earlier model proposed by K. Jain *et al.* [1], therefore, we will describe only the essential component of formulation here. The bandhead energy of 3QP quadruplet is given as:

$$E(K) = E_{qp} + E_{rot.} + E_{irrot.} + E_{resi.} + E_{rpc} + E_{ppc} + E_{pair}$$

Here, E_{pair} is proton and neutron pairing energy and is calculated using four point formulae.

$$\Delta_p = \frac{1}{4}\{B(N,Z-2) - 3B(N,Z-1) + 3B(N,Z) - B(N,Z+1)\}$$

$$\Delta_n = \frac{1}{4}\{B(Z,N-2) - 3B(Z,N-1) + 3B(Z,N) - B(Z,N+1)\}$$

The binding energies used in above equations are taken from latest mass evaluation [2].

The E_{qp} is the total quasiparticle energy and is given by the sum of the three one quasiparticle energies as:

$$E_{qp} = \sum_{i=1}^{3} E_i$$

E_{rot} is the rotational energy and is given by:

$$E_{rot} = \frac{\hbar^2}{2\Im}(I(I+1) - K^2)$$

E_{irrot} is the irrotational energy which is purely the contribution of the three odd particles and is given by:

$$E_{irrot} = \frac{\hbar^2}{2\Im}\left[\begin{array}{l}\left(\sum_{j_1}\left|C_{k_1}^{j_1}\right|^2 j_1(j_1+1) - k_1^2\right) \\ +\left(\sum_{j_2}\left|C_{k_2}^{j_2}\right|^2 j_2(j_2+1) - k_2^2\right) \\ +\left(\sum_{j_3}\left|C_{k_3}^{j_3}\right|^2 j_3(j_3+1) - k_3^2\right)\end{array}\right]$$

Here, $\left|C_{k_1}^{j_1}\right|^2$ are the Nilsson coefficients obtained from the Nilsson model calculations [3] by using the deformation parameters from Moller *et al.* [4]. E_{res} is the energy corresponding to residual interactions and has the same form as used in earlier model [1]. The GM splitting and Newby shifts energies used in calculations of E_{res} are extracted from the available experimental bandhead energies in adjacent even-even and odd-odd nuclei. The rotor-particle coupling term (E_{rpc}) gives a diagonal contribution to the bandhead energy and is given by:

$$E_{rpc}=\delta_{K,\frac{1}{2}}\frac{\hbar^2}{2\Im}(-1)^{I+\frac{1}{2}}\left(I+\frac{1}{2}\right)\times$$
$$\times\begin{Bmatrix}(\delta_{\sigma_{++-}}+\delta_{\sigma_{+-+}})\left(\langle k_1\rho_1|\,j_{1^+}\,|-k_1\rho_1\rangle\delta_{k_1,\frac{1}{2}}\right)\\ +(\delta_{\sigma_{-++}}+\delta_{\sigma_{++-}})\left(\langle k_2\rho_2|\,j_{2^+}\,|-k_2\rho_2\rangle\delta_{k_2,\frac{1}{2}}\right)\\ +(\delta_{\sigma_{+-+}}\delta_{\sigma_{-++}})\left(\langle k_3\rho_3|\,j_{3^+}\,|-k_3\rho_3\rangle\delta_{k_3,\frac{1}{2}}\right)\end{Bmatrix}$$

The contribution of the particle-particle coupling term is given by:

$$E_{ppc}=\delta_{K,\frac{1}{2}}\frac{\hbar^2}{2\Im}\times$$
$$\times\begin{Bmatrix}(\delta_{\sigma_{+-+}}+\delta_{\sigma_{-++}})\begin{pmatrix}\langle k_1\rho_1|\,j_{1^+}\,|-k_1\rho_1\rangle\times\\ \langle k_2\rho_2|\,j_{2^+}\,|-k_2\rho_2\rangle\delta_{k_1,\frac{1}{2}}\delta_{k_2,\frac{1}{2}}\delta_{k_3,\frac{1}{2}}\end{pmatrix}+\\ (\delta_{\sigma_{-++}}+\delta_{\sigma_{++-}})\begin{pmatrix}\langle k_1\rho_1|\,j_{1^+}\,|-k_1\rho_1\rangle\times\\ \langle k_3\rho_3|\,j_{3^+}\,|-k_3\rho_3\rangle\delta_{k_1,\frac{1}{2}}\delta_{k_3,\frac{1}{2}}\delta_{k_2,\frac{1}{2}}\end{pmatrix}+\\ (\delta_{\sigma_{+-+}}+\delta_{\sigma_{++-}})\begin{pmatrix}\langle k_2\rho_2|\,j_{2^+}\,|-k_2\rho_2\rangle\times\\ \langle k_3\rho_3|\,j_{3^+}\,|-k_3\rho_3\rangle\delta_{k_2,\frac{1}{2}}\delta_{k_3,\frac{1}{2}}\delta_{k_1,\frac{1}{2}}\end{pmatrix}\end{Bmatrix}$$

Using above said contributions of various terms, the bandhead energies of members of a 3QP quadruplet can be calculated.

Results and Discussion

In this paper, we present the calculations of bandhead energies of 3QP quadruplets corresponding to four different 3QP configurations observed in ^{175}Lu and ^{177}Lu nuclides. The experimental data for these quadruplets is taken from ENSDF database [5]. The comparison of calculated bandhead energies of 3QP quadruplets corresponding to 7/2[404]$_\pi$⊗7/2[514]$_\nu$⊗5/2[512]$_\nu$ and 7/2[404]$_\pi$⊗7/2[514]$_\nu$⊗1/2[521]$_\nu$ configurations observed in ^{175}Lu, 7/2[404]$_\pi$⊗7/2[514]$_\nu$⊗1/2[510]$_\nu$ and 7/2[404]$_\pi$⊗7/2[514]$_\nu$⊗1/2[521] configurations observed in ^{177}Lu are shown in Fig. 1 and 2, respectively. From these figures, it is clear that, the present model calculations are in much better agreement with experimental data as compared with the results of earlier calculations [1].

On the basis of present calculations, we suggest that, the inclusion of particle-particle coupling (ppc), rotor particle coupling (rpc) and irrotational contributions efficiently improve the calculated bandhead energies of 3QP quadruplets observed in ^{175}Lu and ^{177}Lu nuclides and hence present version of model will be useful for experimentalist in fixing band-heads of a given 3QP quadruplet.

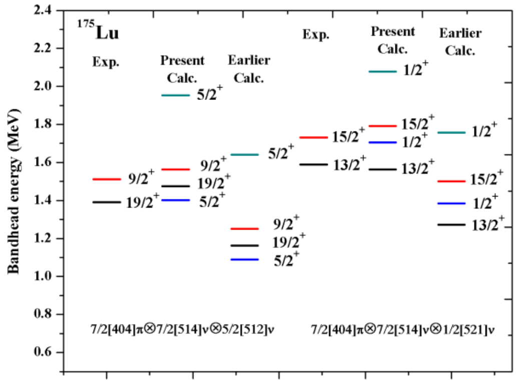

Fig. 1: Comparison of calculated bandhead energies of 3QP quadruplets observed in ^{175}Lu using present and earlier model [1].

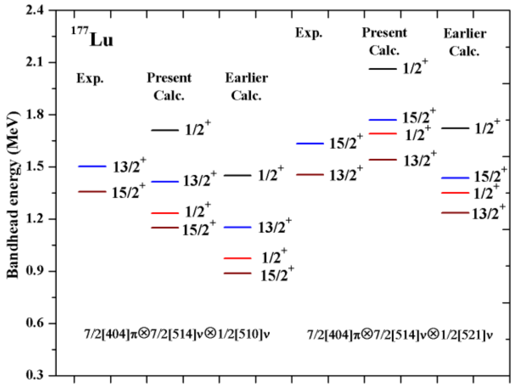

Fig. 2: Same as Fig. 1 but for ^{177}Lu.

Acknowledgement

The financial support from DAE-BRNS, Govt. of India (Sanction No. 36(6)/14/60/2016-BRNS/36145) is gratefully acknowledged.

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