Neutron-Carbon scattering by Manningï Rosen potential

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Introduction

For most of the physical situations the Schrödinger equation is exactly solvable for a fixed value of ℓ and all values of energy. But there are only few potentials like square well, harmonic oscillator, Coulomb etc. which are exactly solvable for all partial waves and energies. However, the study of exponential type potential like Manningï Rosen potential [1], applicable for molecular systems, cannot be solved exactly for all partial waves and at all energies. It is exactly solvable for s-wave only. In the following we shall construct regular solution to the Manningï Rosen potential [1] via the differential equation approach to the problem and find the associated Jost function to compute scattering phase shifts for nucleon-nucleus system. The S-wave Schrödinger equation for the Manning-Rosen potential is written as

$$\left[\frac{d^2}{dr^2}+k^2-\frac{b^{-2}\alpha(\alpha-1)}{(1-e^{-r/b})^2}e^{-2r/b}+\frac{b^{-2}Ae^{-r/b}}{1-e^{-r/b}}\right]\varphi(k,r)=0 \quad . \tag{1}$$

Here A, b and α are three adjustable parameters. The wave function $\varphi(k,r)$ satisfies the regular boundary condition. Under the transformation

$$\varphi(k,r)=b^{\alpha}(1-e^{-r/b})^{\alpha}e^{ikr}g(k,r) \tag{2}$$

equation (1) becomes

$$b^2e^{r/b}(1-e^{-r/b})g''(k,r)+2b\times\left[\alpha+ikb\,e^{r/b}(1-e^{-r/b})\right]g'(k,r)+\left[2ikb\alpha-\alpha+A\right]g(k,r)=0 \quad . \tag{3}$$

Substituting the variable $(1-e^{-r/b})=z$ the above equation leads to

$$z(1-z)\frac{d^2g}{dz^2}+\left[2\alpha-(1+2\alpha-2ikb)z\right]\frac{dg}{dz}-(\alpha-A-2ikb\alpha)\,g=0. \tag{4}$$

Comparing Eq. (4) with the standard differential equation for Gaussian Hypergeometric function [2, 3]

$$z(1-z)\frac{d^2g}{dz^2}+\left[c'-(1+a'+b')z\right]\frac{dg}{dz}-a'b'g=0 \tag{5}$$

we get

$$g(k,z)={}_2F_1\left(a',b';c';z\right) . \tag{6}$$

Therefore, from Eqs. (2) and (6) the regular solution $\varphi(k,r)$ with $\alpha=\alpha+1$ is expressed as

$$\varphi(k,r)=b^{\alpha+1}(1-e^{-r/b})^{\alpha+1}e^{ikr}\times{}_2F_1\left(A',B';C';1-e^{-r/b}\right) , \tag{7}$$

where

$$A'=\alpha+1-ikb+\left(\alpha^2-k^2b^2+\alpha+A\right)^{1/2}, \tag{8a}$$

$$B'=\alpha+1-ikb-\left(\alpha^2-k^2b^2+\alpha+A\right)^{1/2} \tag{8b}$$

$$C'=2\alpha+2 . \tag{8c}$$

Using the analytic continuation formula of Gaussian hypergeometric function [2. 3]

$${}_2F_1(a,b;c;z)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\times{}_2F_1(a,b;a+b-c+1;1-z)+(1-z)^{c-a-b}\times\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}{}_2F_1(c-a,c-b;c-a-b+1;1-z) \tag{9}$$

and

$${}_2F_1\left(a,b;c;z\right)=(1-z)^{c-a-b}{}_2F_1\left(c-a,c-b;c;z\right) \tag{10}$$

equation (7) leads to

$$\varphi(k,r)=\frac{1}{2ik}\left[\frac{\Gamma(2\alpha+2)\Gamma(1+2ikb)}{\Gamma(A'^*)\Gamma(B'^*)}\frac{b^{\alpha}e^{ikr}}{(1-e^{-r/b})^{\alpha}}\times{}_2F_1\left(A'-2\alpha-1,B'-2\alpha-1;\,1-2ikb;e^{-r/b}\right)-\frac{\Gamma(2\alpha+2)\Gamma(1-2ikb)}{\Gamma(A')\Gamma(B')}b^{\alpha}(1-e^{-r/b})^{-\alpha}e^{-ikr}\times{}_2F_1\left(A'^*-2\alpha-1,B'^*-2\alpha-1;\,1+2ikb;e^{-r/b}\right)\right]. \tag{11}$$

There exists a relation between regular and irregular solutions [4]

$$\varphi(k,r)=\frac{1}{2ik}\left[\Im_{-}(k)f_{+}(k,r)-\Im_{+}(k)f_{-}(k,r)\right], \tag{12}$$

where the Jost function $\Im_{+}(k)=\left(\Im_{-}(k)\right)^{*}$. On comparing Eqs. (11) and (12) the Jost solution and the Jost function are identified as

$$f_{+}(k,r)=(1-e^{-r/b})^{-\alpha}\,e^{ikr}\times \; {}_2F_1\left(A'-2\alpha-1, B'-2\alpha-1;\, 1-2ikb; e^{-r/b}\right) \tag{13}$$

and

$$\Im_{+}(k)=b^{\alpha}\,\frac{\Gamma(2\alpha+2)\Gamma(1-2ikb)}{\Gamma(A')\Gamma(B')}\,. \tag{14}$$

By exploiting the integral representation of the Jost function [4, 5]

$$\Im(k)=1+\int_{0}^{\infty} dr\,V(r)e^{ikr}\varphi(k,r) \tag{15}$$

we get the same expression as given in Eq. (14). As the phase of the Jost function is the negative of the scattering phase shift we shall compute neutron-Carbon elastic scattering phase shifts by using Eq. (14). For n-C^{12} system the parameters are $b=0.075\,fm$, $A=1.005$ & $\alpha=-1.0$ which support correct binding energy.

Table 1: Scattering phase shifts for the n-C^{12} system.

E_{Lab} (MeV)	ŭ$^{1/2(+)}$ (degree)	Ref. [6]
1.083	19.74	é é
2.05	26.12	é é
3.03	30.54	é é
4.00	33.92	é é
5.09	36.89	é é
6.07	39.09	é é
7.50	41.70	é é
8.91	43.82	42.5
9.0	43.96	43.0
9.2	44.25	43.4
9.35	44.40	44.9
9.5	44.68	45.6
10.0	45.34	45.7
10.22	45.59	47.4
10.5	45.84	48.6

As observed that our computed phase shifts for the neutron-Carbon system by the Jost function method are in close agreement with those of Tornow et al. [6] in the entire energy range under consideration. From Table 1 it is clear that our potential model is quite capable of producing nucleon-nucleus elastic scattering phase shifts. However, the Manningï Rosen potential [1] may also be used to calculate scattering phase shifts for nucleon-nucleon, alpha-nucleon and nucleus-nucleus systems. It is well known that zeros of the Jost function in the upper half of the complex momentum space reproduces the binding energies of the systems concerned. Thus, for bound system one can extract information regarding the bound states. Within this formalism one is able to treat both bound and scattering states respectively. We hope, our treatment may be of quite promising and interesting to nuclear physicists.

References

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