# Semiclassical level density parameter with collective enhancement

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## Introduction

Nuclear level density play an important role in describing the nuclear structure, reaction dynamics and the nuclear astrophysical phenomenon [1-3]. It is also essential in deriving the thermodynamic properties such as entropy, excitation energy and specific heat etc. of an excited atomic nucleus. The analytical formula based on non-interacting Fermi gas was first proposed by Bethe [4]. This approach serves as a building block in the widely used phenomenological models such as constant temperature model(CTM), back-shifted Fermi gas model(BSFGM) and the Gilbert-Cameron model(CGCM).

This work aims towards the calculation of level density parameter(used as input in the level density calculations) using semiclassical trace formula for deformed nuclei with spin-orbit interactions [5-6]. The collective effects such as vibrational and rotational, characteristic of coherent motion of large number of nucleons have been taken care through the use of recently proposed level density parameter formula [7].

## Semiclassical calculation of collective effects in the level density parameter

The semiclassical expression of the average single-particle level density for axially symmetric harmonic oscillator potential with spin-orbit interactions is given as [5-6]:

$$\begin{aligned}\tilde{g}(E) &= \frac{E^2}{2a\hbar\omega_0^{0\,3}}\left[\,1+\kappa^2 b\right] \\ &- \frac{b}{24a\hbar\omega_0^0}\left[\,1+\kappa^2\frac{\left(\,b^2+2c\right)}{b}\right] \\ &+ \frac{E\kappa^3 c}{3a\hbar\omega_0^{0\,2}}+\mathcal{O}(\hbar^4\kappa^4)+.... \qquad (1)\end{aligned}$$

where, $a=\hbar{\omega_\perp}^2\hbar\omega_z, b=2\hbar{\omega_\perp}^2+\hbar{\omega_z}^2, c=\hbar{\omega_\perp}^4+2\hbar{\omega_\perp}^2\hbar{\omega_z}^2$. The value of $\kappa$ in units of $(\hbar\omega_0^0)^{-1}$ is taken from [8]. The oscillator frequencies are defined in terms of the deformation parameter $\epsilon$ as:

$$\omega_\perp=\omega_0(\epsilon)\left(\,1+\frac{1}{3}\epsilon\right),\quad \omega_z=\omega_0(\epsilon)\left(\,1-\frac{2}{3}\epsilon\right)$$

$\omega_0(\epsilon)$ is determined from the condition of incompressibility of nuclear matter and is given as:

$$\omega_0(\epsilon)=\omega_0^0\left(\,1+\frac{\epsilon^2}{9}\right) \qquad (2)$$

The Nilsson deformation parameter $\epsilon$ is defined in terms of quadrupole deformation $\beta_2$ as, $\epsilon\approx 0.95\beta_2$. and the spacing between the oscillator levels is chosen as [8]:

$$\hbar\omega_0^0(n,p)=\frac{41}{A^{\frac{1}{3}}}\left(\,1\pm\frac{N-Z}{A}\right)^{\frac{1}{3}}.$$

The level density parameter with the inclusion of collective enhancement is given as [7]:

$$a(U) = \tilde{a}\left(1+A_C\frac{S_n}{U}\frac{\exp(-|U-E_0|{\sigma_c'}^3)}{{\sigma_c'}^3}\right) \qquad (3)$$

where, $S_n$ is the neutron separation energy, $A_C$ is the collective amplitude defined in terms of microscopic shape-dependent shell corrections:

$$\begin{aligned}A_C &= (B_{LDM}-B_{EXPT}-E\theta^2)\frac{\tau_C}{Sinh(\tau_C)} \\ \tau_C &= \frac{2\pi^2 T_c}{\hbar\omega} \qquad (4)\end{aligned}$$

where, $B_{LDM}$[9] and $B_{EXPT}$[10] refers to the liquid drop model and experimental binding energies respectively. $E\theta^2$ is the deformation energy [7]. $T_c$ is the nucleus critical temperature at neutron separation energy $S_n$ defined as, $T_c = \sqrt{\frac{S_n}{\tilde{a}}}$ and $E_0$ in expression(3) is the energy of the first phonon state usually taken as $0.2\hbar\omega$ and $\hbar\omega = \frac{41}{A^{1/3}}$.

The spin cut-off parameter $\sigma_c^{'}$ in expression(3) refers to $\sigma_c/\tilde{a}$ [7]. The main ingredient in the level density parameter calculation comes from it's smooth value $\tilde{a}$, which is obtained from the semiclassical single-particle level density as:

$$\tilde{a} = \frac{\pi^2}{6}(g_p(E_F^p + \delta) + g_n(E_F^n + \delta)),$$
$$\delta = S(N,Z) - \Delta \qquad (5)$$

where, $S(N,Z)$ is the shell correction energy [10] and $\Delta = \frac{12}{\sqrt{A}}$ for even-even nuclei and $E_F^{p,n}$ refers to the Fermi energies.

## Results and Discussion

The results for level density parameter evaluated from eq. (3) as a function of excitation energy $U$ for $^{188}Os$ is plotted in Fig. 1:

It is observed that 'a' initially increases with

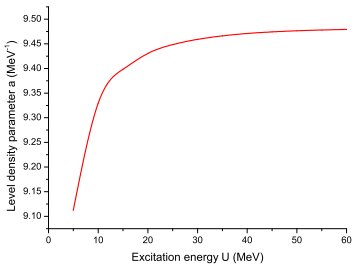


Figure 1: Level density parameter 'a' as a function of excitation energy $U$ for $^{188}Os$.

'U' and then attains a constant value of $\simeq 9.47$ $MeV^{-1}$ at higher excitation energies. The results for level density parameter evaluated from eq. (3) as a function of excitation energy $U$ for $^{204}Pb$ is plotted in Fig. 2:

In case of $^{204}Pb$, a constant value of $\simeq$

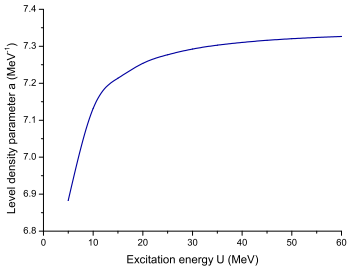


Figure 2: Level density parameter 'a' as a function of excitation energy $U$ for $^{204}Pb$.

$7.32MeV^{-1}$ is attained at higher excitations.

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