Comparison of ab formula and Soft rotor formula

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For the nuclei of the medium mass region, the level energies in ground state collective band of even *Z*-even *N* nuclei, with spin I=0, 2, 4, 6,..deviate from the ideal rotor formula [1]. Several empirical energy relations have been used to fit the level energies with varying degree of success in different regions of the nuclide chart.

For example, the Bohr Mottelson formula in powers of $X=I(I+1)$ having two or more terms is suitable for good rotors. The extended form of three term Rotation vibration interaction relation [2]

$$E= a\, I + b\, I(I+1) + c\, I^2(I+1), \quad (1)$$

was used to study shape transitional nuclei in [2].

The 2 parameter *ab* formula [3], is expressed as

$$E=a\,[(1+bI(I+1))^{1/2} - 1] \quad (2)$$

the assumption that the moment of inertia MoI increases with level energy. The 2-parameter, Soft Rotor Formula (SRF) [4] assumes the linear dependence of MoI θ (I) on spin I

$$E(I)=\hbar^2 I(I+1)/[2\theta_0(1+\alpha I) \quad (3)$$

Here we make a comparison of the two 2-parameter formulae (2) and (3) using experimental energies from NNDC website compilation for the deformed nuclei (in table 1) having energy ratio $R_{4/2}$ from 3.30 to 3.0. The parameter evaluation is done in predictive mode for both formulae.

Result: As the energy ratio $R_{4/2}$ decreases 3.30 to 3.00, in the ab formula the parameter 'a' decreases by a factor of 7 and 'b' increases by a factor of 10 and the relative error sigma (in %) varies by a factor of 30. Moreover, it deteriorates further for lower $R_{4/2}$. In contrast, in SRF the relative error σ (in %) varies with $R_{4/2}$ by only a small amount (in Table 2).

Conclusion: In the ab formula, the variation of parameters a, b with decreasing $R_{4/2}$ is too fast, and the relative error sigma increases too fast. On the other hand, in SRF the variation of σ with $R_{4/2}$ is rather small. Moreover, SRF yields E(I) in agreement with experiment for even lower $R_{4/2}$.

References

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Table 1. Comparison of experimental energies in SRF and ab formulae.

Nuclei	Energy	E(2)	E(4)	E(6)	E(8)	E(10)	E(12)
Dy164	Expt	73.4	242.2	501.3	843.7	1261.3	1745.9
	NSM	73.4	242.2	503.5	854.7	1410.5	1833.7
	Ab	73.4	242.2	500.9	841.8	1256.0	1734.6
Hf172	Expt	95.2	309.3	628.3	1037.5	1521.2	2064.7
	NSM	95.2	309.3	633.1	1058.6	1722.0	2238.6
	Ab	95.2	309.3	625.5	1024.8	1488.8	2002.6
Gd156	Expt	89.0	288.2	584.7	965.1	1416.1	1924.5
	NSM	89.0	288.2	588.4	981.5	1592.8	2070.7
	Ab	89.0	288.2	580.9	947.9	1372.0	1839.2
Hf170	Expt	100.8	322.0	642.6	1043.0	1504.2	2015.9
	NSM	100.8	322.0	649.1	1069.9	1717.1	2232.3
	Ab	100.8	322.0	638.1	1023.7	1458.5	1927.9
W172	Expt	123.2	377.1	727.6	1146.8	1617.3	2131.0
	NSM	123.2	377.1	732.1	1166.8	1817.0	2362.2
	Ab	123.2	377.1	714.4	1101.0	1517.2	1951.9
Hf166	Expt	158.6	470.5	897.2	1406.4	1971.9	2565.8
	NSM	158.6	470.5	890.1	1388.3	2122.4	2759.2
	Ab	158.6	470.5	866.9	1307.6	1773.1	2253.1

Table 2. Parameters of the energy formulae.

Isotope	A	b×100	RMSD	Sigma %	$R_{4/2}$	Theta	alpha	RMSD	Sigma %
Dy164	8472	0.29	4.2	0.27	3.30	40.5	0.0051	36	2.28
Hf172	3985	0.81	36	1.95	3.25	30.7	0.0136	68	3.7
Gd156	3378	0.89	47	2.8	3.24	32.7	0.0069	53	3.0
Hf170	2491	1.38	51	2.88	3.19	28.5	0.0227	78	4.3
W172	1399	3.06	98	5.3	3.06	22	0.049	50	2.67
Hf166	1180	4.78	193	8.9	2.97	17	0.071	19	1.1