# Correlation between symmetry energy and deformed magic number in some rare earth nuclei

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## Introduction

The development of radioactive ion beam facilities provide the path for the exploration of nuclear structure of exotic nuclei across the drip lines and to investigate the effect of neutron-proton asymmetry on the strong nuclear interaction. It is intriguing to examine that whether the classical magic character holds even far from the stability line. Several investigations have shown the evolution of new shell gaps in exotic nuclei along with quenching of classical magic numbers in light mass region [1–3]. While few such studies are available in medium mass exotic nuclei [4, 5].

The exotic nuclei with large imbalance of neutron-proton asymmetry entail significant amount of symmetry energy. The knowledge about symmetry energy is essential to understand the structure of exotic nuclei and neutrons stars. It is noted that some kinks are seen in the symmetry energy of the isotopic chains of light to heavy nuclei at classical magic numbers depicting the influence of shell closure on symmetry energy [6]. Here, we investigate the symmetry energy and its volume and surface components in isotopic chains of rare earth Nd and Sm nuclei, within coherent density fluctuation model (CDFM) [7, 8], to explore the correlation, if any, between symmetry energy and deformed magic number in these nuclei.

## Formalism

The expression for symmetry energy given in the liquid droplet model, an extension of the Bethe-Weizsäcker liquid drop model, incorporating the volume asymmetry ($S_V$) and surface asymmetry ($S_S$) can be written in the form S$\times(N-Z)^2$/A (see Ref. [9–11]), where

$$S = \frac{S_V}{1+\frac{1}{\kappa A^{1/3}}}; \kappa = S_V/S_S \qquad (1)$$

From above Eq. (1) we get

$$S_V = S\left(1+\frac{1}{\kappa A^{1/3}}\right); S_S = \frac{S}{\kappa}\left(1+\frac{1}{\kappa A^{1/3}}\right) \qquad (2)$$

The symmetry energy S within CDFM is calculated by

$$S = \int_0^\infty dx|F(x)|^2 S[\rho(x)], \qquad (3)$$

where the weight function $|F(x)|^2$ depends on the density distribution

$$|F(x)|^2 = -\left(\frac{1}{\rho_0(x)}\frac{d\rho(r)}{dr}\right)_{r=x}. \qquad (4)$$

The axially deformed densities are taken from RMF model [12] with NL3 and IOPB-I parameter sets and are converted into spherical equivalent densities by following the method of Ref. [13] for further use in Eq. (4). The

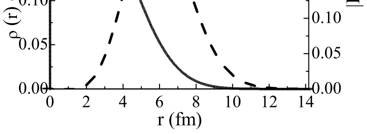


FIG. 1: The density ($\rho$) and weight function $\mid F(x) \mid^2$ of $^{144}Sm$ corresponding to IOPB-I parameter set.

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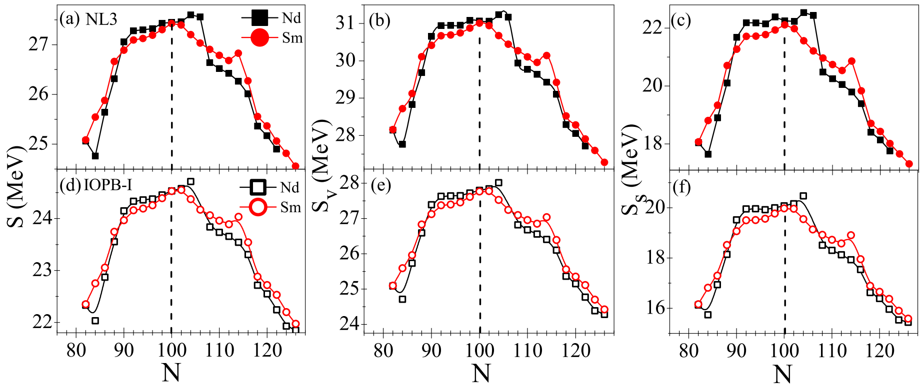


FIG. 2: The variation of symmetry energy (S), volume ($S_V$) and surface symmetry energy ($S_S$) with neutron number for Nd and Sm nuclei with (a, b, c) NL3 and (d, e, f) IOPB-I parameter sets.

expression for ratio $\kappa$ within CDFM following [14] is,

$$\kappa = \frac{3}{R\rho_0}\int_0^\infty dx|F(x)|^2 x\rho_0(x)\left[\left(\frac{S(\rho_0)}{S(\rho(x))}\right) - 1\right] \quad (5)$$

where density dependence of symmetry energy [14, 15] given by

$$S[\rho(x)] = S_0\,(\rho(x)/\rho_0)^\gamma \quad (6)$$

Here, $\gamma = 0.3$ is used in reference to [8].

## Results and discussion

Fig. 1 shows the spherical equivalent density ($\rho$) and weight function $\mid F(x) \mid^2$ of $^{144}Sm$ as a representative case, with IOPB-I parameter set. It is noted that weight function has a bell shape with maxima near the surface part of the density. It shows that surface part of density contribute significantly in the symmetry energy calculations and hence these studied quantities are termed as surface properties.

Using the calculated weight functions, the symmetry energy S and volume ($S_V$) and surface symmetry energy ($S_S$) components are shown in Fig. 2 for Nd and Sm nuclei for NL3 (upper panel) and IOPB-I (lower panel) parameter sets. A rise and fall trend with a peak at neutron number N = 100 is seen in Fig. 2. This peak is a manifestation of deformed shell closure/magic number in Nd and Sm nuclei. This result is in consonance with earlier study by one of us, demonstrating the stability of $^{162}Sm$ nucleus [4], which had been experimentally conformed by Patel et al. in 2014 reporting the existence of deformed shell closure at N = 100 in $^{162}Sm$ and $^{164}Gd$ isotones [5]. This result presents a correlation between symmetry energy and deformed magic number both being neutron-proton asymmetry dependent. Moreover, this result is of considerable importance since the N = 100 nuclei due to stability can act as a waiting point in the nucleosynthesis via $r$-process.

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