

## Effect of shell corrections on the spontaneous fission using the quantum mechanical fragmentation theory

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### Introduction

Nuclear Physics has passed by so many milestones of which the radioactivity has been proved to be the most fundamental step for the large scale researches that have been done so far. Radioactive nature is exhibited by unstable nuclei *i.e.* the nuclei far off from the stability line. These unstable nuclei can achieve stability only by decaying into their daughter fragments. Earlier,  $\alpha$ -decay used to be considered as the most prominent and usual decay mode in case of heavy and superheavy mass region. In 1984, one more profound decay mode came into picture *i.e.* the cluster radioactivity in which daughter fragment having mass greater than  $\alpha$ -particle and less than the mass of lightest fission fragment is expelled out [1]. Besides the two decay modes discussed above, one more beneficial decay mode was appreciated subject to heavier mass region *i.e.* spontaneous fission while exploring the heavy particle radioactivity (HPR) [2].

Generally heavier mass nuclei undergo spontaneous fission as their binding energies are low and their activation energies are high. As we move far off the  $\beta$ -stability line more and more number of shells are added and therefore shell corrections start playing a vital role when calculating the binding energies and hence the half lives. Various theoretical explorations have been done subjecting spontaneous fission. The mass asymmetries of the fission fragments are considered as ground for finding out various parameters and for the calculations of SF half lives. One of the two fragments has spherically closed shell structure or nearby shell closure, is an interesting fact to be noted.

### The Preformed Cluster-decay Model (PCM)

Different theoretical models are available for the study of radioactivity. In the present work, the

preformed cluster-decay model (PCM) [3] based on quantum mechanical fragmentation theory (QMFT) is employed for the study of spontaneous fission. In PCM, the clusters or fragments are assumed to be preborn in their parent nucleus with different probability of preformation and their ejection takes place via tunneling the barrier (quantum tunneling). This model was found to be very apt in calculating many parameters like fragmentation potential, preformation probability, binding energies and half lives of the parent nuclei establishing the concepts of QMFT. The Schrödinger eq. in terms of mass parameter is given by

$$\left\{ \frac{\hbar^2}{2\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} + V_R(\eta, T) \right\} \psi^v(\eta) = E^v \psi^v(\eta) \quad (1)$$

Here,  $\eta = (A_1 - A_2) / (A_1 + A_2)$ .  $\psi^v(\eta)$  represents the vibrational states. Only the ground state solution ( $v = 0$ ) is relevant for spontaneous fission. On solving Eq. (1), we get,  $P_0(A_i) \equiv |\psi^v(\eta)(A_i)|^2$ .

The first turning point  $R_0 = R_1 + \Delta R$ ,  $R_1 = R_1 + R_2$ , where  $\Delta R$  is the neck length parameter. The fragmentation potential  $V_R(\eta)$  in eq. (1) is given by:

$$V_R(\eta) = - \sum_{i=1}^2 [B(A_i, Z_i)] + V_c(R, Z_i) + V_p(R, A_i) + V_\ell(R, A_i) \quad (2)$$

In eq. (2) second term represents the Coulomb interaction.  $V_p$  and  $V_\ell$  represent the nuclear proximity and centrifugal potentials respectively. The first term in eq. (2) is written is:

$$B(A_i, Z_i) = \sum_{i=1}^2 [V_{LDM}(A_i, Z_i)] + \sum_{i=1}^2 [\delta U_i] \quad (3)$$

Myers and Swiatecki [4] proposed a two part approach. The first term in the eq. (3) is the macroscopic term which was defined using Bethe-Weizsacker formula based on the liquid drop model. The second term leads us to the shell correction energy which had to be added to macroscopic binding energy to incorporate shell

effects as shell structure changes drastically as we move far off stability line. The microscopic term in eq. (3) is given by [5]:

$$\delta U = C \left[ \frac{F(N) + F(Z)}{(A/2)^{2/3}} - cA^{1/3} \right] \quad (4)$$

where,

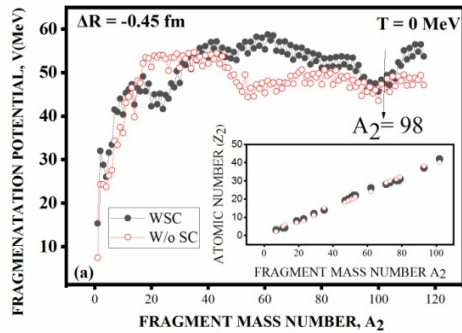
$$F(X) = \frac{3}{5} \left( \frac{M_i^{5/3} - M_{i-1}^{5/3}}{M_i - M_{i-1}} \right) (X - M_{i-1}) - \frac{3}{5} (X^{5/3} - M_{i-1}^{5/3}) \quad (5)$$

In Eq. (4)  $X=Z$  or  $N$ ,  $M_{i-1} < X < M_i$ .  $M_i$  is the magic number (2, 8, 20, 28, 50, 82, and 126).

The tunneling probability ( $P$ ) is calculated using WKB approximation. The decay constant and hence decay half life is then given by  $\lambda = \nu_0 P_0 P$ ,  $T_{1/2} = \ln 2 / \lambda$ . For details see Ref. [3].

### Results and Discussions

The fragmentation potential (shown for  $^{232}\text{U}$  in Fig.1) shows positive as well as negative effects when shell corrections are switched off but in most of the mass region fragmentation potential decreases when shell corrections are not included. The neck-length is taken from the Ref. [6] obtained there by fitting the actual experimental half-lives.

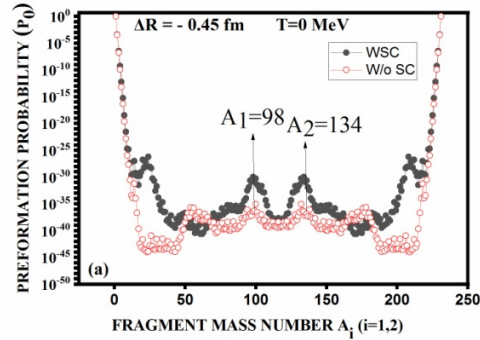


**Figure 1** The deviation in fragmentation potential with (WSC) and without shell correction energy (W/o SC) is plotted against fragment mass number ( $A_2$ ) for  $^{232}\text{U}$ . The corresponding variation of atomic number under the effect of shell corrections is depicted in the inset.

The fragmentation potential has increment in some region in absence of shell correction. It increases for  $A_2=15-36$ .

The inset in the Fig. 1 shows the change in atomic mass number of fragments while including and excluding the shell correction energy. It has been observed that after switching off shell correction energy the atomic number of

few fragment changes and magicity of some nuclei vanishes. Hence shell correction affects the fragmentation potential as well as atomic number of some of the fragments.



**Figure 2** The variation in preformation probability ( $P_0$ ) with (WSC) and without shell correction energy (W/o SC) is plotted against fragment mass number  $A_i$  for  $^{232}\text{U}$ .

Clear differences are observed in the preformation probability (Fig. 2) because of the corresponding variations in the fragmentation potential which depends on binding energy and hence shell correction energy. Also, the preformation probability is a relative factor, and its magnitude is dependent upon the relative contribution of all the decaying fragments and not only on the individual decay channel due to the collective clusterization procedure followed in the PCM. The preformation probability decreases when shell corrections are switched off for maximum number of fragments. The effects of shell correction energy of all the fragments are clearly reflected in the preformation factor here. On calculating the half lives for spontaneous fission, it is observed that it increases when we do not incorporate the shell corrections.

### References

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