Collapse of White Dwarf Stars in Strong Quantizing Magnetic Field- A Thomas-Fermi Approach

Sanchita Das,[1,*] Sutapa Ghosh,[2] and Somenath Chakrabarty[1]
[1]*Department of Physics, Visva-Bharati, Santiniketan, India 731235*
[2]*Taki Govt. College, Taki, North 24 Parganas, India 743 429*

1. Introduction

It is strongly believed that white dwarfs are formed at the late stage of chemical evolution of a star of mass ranging from $M_\odot$ to $8M_\odot$ [3], where $M_\odot$ is the mass of sun. At the stage when the electron gas at the core region of the progenitor becomes degenerate, there will be no further thermo-nuclear reaction at the central region. Depending on the mass of the original star, the central region will be filled up with fully ionized massive atoms like carbon, oxygen, etc., in the background of degenerate electron gas. For low mass stars, at this stage the central region is mainly consisting of helium ions and degenerate electron gas. Since the thermo-nuclear reactions are stopped at this stage, this class of white dwarfs are extremely metal poor. Although there is no more thermo-nuclear reaction at the degenerate core region. As a consequence the outer region will expand in the radially outward direction, whereas the core region, where the inward force due to the self-gravity of the object is balanced by the degeneracy pressure of electron gas remains in mechanical equilibrium. Therefore a dynamical mechanical instability will be developed between the crust region and the core of the system. The ejected material will form a nebula like object surrounding the central compact object of mass $\sim 1.41M_\odot$, known as Chandrasekhar mass [4]. This compact stellar object is called the white dwarf. Since energy of the electron gas in such ultra dense matter is high enough, they behave as a degenerate gas and never form Cooper pairs. In the present article, we have shown that if the Landau levels of the electrons are populated, which occurs if the magnetic field strength is greater than a critical, the object becomes gravitationally unstable. The critical value is obtained from the condition that for the quantum mechanical effect of strong magnetic field, the cyclotron quantum $\omega_c = eB/m_e \geq m_e$ [7] (the critical field strength for electron $B_c^{(e)} \approx 4.4 \times 10^{13}$G).

2. Basic Formalism

We have considered a white dwarf star, with an envelop of fully ionized helium and hydrogen. All these positively charged heavy ions are immersed in a background negatively charged degenerate electron gas. The mass of white dwarf is $\sim 1.41M_\odot$ contributed by the rest masses of the heavy ions. It is well known that degenerate electron gas balances the inward force due to the self-gravitation of white dwarf stars [10]. It can very easily be shown that when the Landau levels of the electron gas inside t he white dwarfs are populated, the degeneracy pressure of electron gas drops by at least two orders of magnitude [7–9]. In fig.(1) we have plotted the electron degeneracy pressure for the nan-magnetized case, indicated by the curve a and in presence strong quantizing magnetic field is indicated by curve b. Along x-axis we have plotted $\alpha = n/n_0$, where $n_0 = 0.17\text{fm}^{-3}$, the normal nuclear density and n is the density of electron gas inside the white dwarf stars. Along left side y-axis we have plotted the electron degeneracy pressure for the non-magnetized case, where as right side y-axis we have plotted the same quantity for the magnetized case. The strength of magnetic field is assumed to be 10^{16}G. The electron degeneracy pressure here drops by at least ten times. The drop will be more with the increase in magnetic field

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[*] Electronic address: `sanchita.vbphys@gmail.com`

strength. Of course beyond 10^{19}G, the magnetic pressure dominates, which is positive in nature and the star becomes magnetically unstable [6]. With the decrease in degeneracy pressure of electron gas the star will collapse, the electrons inside the star will be pushed closer to the fully ionized heavy ions like carbon, Oxygen, sulphur etc. These electrons will be absorbed by the nuclei through inverse β-decay and produce more and more neutron rich nuclei. Finally the neutron richness may become so high that the nuclei will no longer remain stable and melt to form a homogeneous mixture of neutron-proton-electron degenerate matter. This is the transition from a white dwarf to a neutron star.

3. Conclusion

We conclude that if by some means the magnetic field of a white dwarf exceed the critical value to populate Landau levels of the electrons or the white dwarf is born with magnetic field strength greater that the critical value, it will collapse and ultimately, when the radius becomes less than the corresponding Schwarzschild radius, it will become a black hole. The main reason for collapse is because of huge reduction in degenerate electron pressure. It has already been studied that the equation of state of electron gas becomes softer in presence of strong magnetic field [8, 9]. Since the degenerate electron gas pressure is holding the object against the inward pull because of its self-gravity, therefore when the equation of state becomes softer, the gravitational force will dominate and causes the collapse. Exactly like the gamma ray bursts which may occur because of magnetic collapse of neutron stars, which are sometime assumed to be one of the viable candidates for gamma ray bursts [10], in this case case also it is possible to have the emission of energy because of the loss of gravitational energy. Of course it will not be of that much violent.

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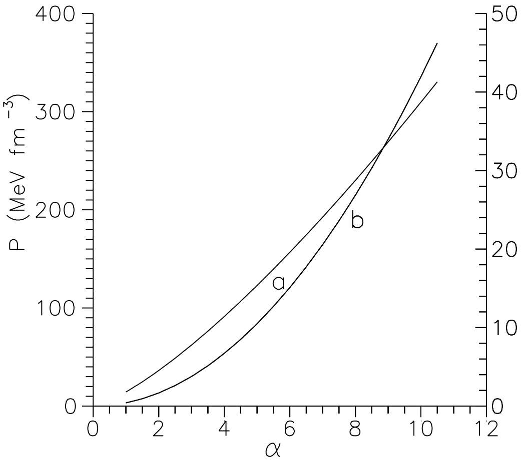

FIG. 1: The variation of degenerate pressure with the density of matter. Curve ia is for $B = 0$, whereas curve b is for $B = 0$

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