The r-mode instability of the neutron star matter under the influence of URCA reactions

T. R. Routray[1,*], S. P. Pattnaik[1,4], X. Viñas[2], D. N. Basu[3], M.Centelles[2] and K. Madhuri[1,5]

[1]*School of Physics, Sambalpur University, Jyotivihar-768 019, India.*
[2]*Departament de FísicaQuànticaiAstrofísica and Institut de CiènciesdelCosmos(ICCUB), Facultat de Física, Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain*
[3]*Variable Energy Cyclotron Center, 1/AF Bidhan Nagar, Kolkata, 700064, India*
[4]*G.M. Jr. College, Sambalpur, Odisha-768004, India*
[5]*Women's Jr. College, Sambalpur, Odisha-768001, India*
** email: trr1@rediffmail.com*

Introduction

The r-mode oscillation in pulsar Neutron stars (NSs) is theoretically proposed to be continuous source of Gravitational wave (GW) emitted by these pulsars. There are different types of instabilities which can operate in a NS. Among them, the so-called r-mode instability, a toroidal mode of oscillation whose restoring force is the Coriolis force. [1,2]. The resulting instability in the pulsar star due to continuous emission of GW is counteracted due to the viscous effects in the NS. The r-mode oscillation evolves with a time dependence of $e^{i\omega t-t/\tau}$, where ω is the real part of the r-mode given by $\omega=\frac{(l-1)(l+2)}{(l+1)}\Omega$, where Ω is the angular velocity of the pulsar. Contribution to the r-mode comes from the emission of GW that tries to render the r-mode unstable and the viscous effects that counter balances the instability due to GW emission. So $\frac{1}{\tau}$ can be written as $\frac{1}{\tau}=\frac{1}{\tau_G}+\frac{1}{\tau_{vis}}$, where τ_G is the gravitational time-scale and $\frac{1}{\tau_{vis}}$ is the sum of the reciprocal of all the different viscous time-scales. The viscous effect considered in this work are bulk and shear viscosities and the viscous effect coming from the crust-core boundary layer, $\frac{1}{\tau_{vis}}=\frac{1}{\tau_{BV}}+\frac{1}{\tau_{sV}}+\frac{1}{\tau_{SE}}$, where τ_{BV} and τ_{SV} are bulk and shear time-scales of the core whereas τ_{SE} is the shear viscous time-scales of the crust-core boundary layer. It is to be noted here that out of all r-modes l=m=2 r-mode is important. The standard analytical expressions for τ_G, τ_{BV}, τ_{SV} and τ_{SE} are cited in ref [3]. From those expressions it can be seen that $\frac{1}{\tau}$ is a function of r-mode angular frequency ω and temperature T. The r-mode is stable for $\frac{1}{\tau} > 0$ and it is unstable for $\frac{1}{\tau} < 0$. So the instability boundary i.e. the critical frequency (ν_c) as a function of T, can be obtained from the condition $\frac{1}{\tau} = 0$. This has been shown in reference [3] for an approximate expression of the bulk viscosity ζ for different equation of state (EOS) constructed from the finite range simple effective interaction (SEI) for different value of slope parameter $L(\rho_0)$.

In this work the instability boundary is calculated taking the contribution of URCA reaction into bulk viscosity ζ of the non-superfluid (npeμ) matter and is compared with the ν_c obtained for an approximate expression of ζ with the EOS of SEI.

.

Formalism

The URCA reactions are of two types, direct URCA (DURCA) and modified URCA (MURCA) reactions. DURCA is a sequence of two reactions,

$n \rightarrow p+l+\bar{\nu}_l$, $p+l \rightarrow n+\bar{\nu}_l$ ...(1)

where, lepton l is either e or μ and ν_l is the associated neutrino. DURCA is subject to the condition $p_n \leq p_p+p_l$, where p_n and p_p are the momentum of the neutron and proton, respectively and p_l is the momentum of the lepton, either e or μ. In MURCA processes an additional nucleon required to conserve momentum,

$n+N \rightarrow p+N+l+\bar{\nu}_l$, $p+N+l \rightarrow n+N+\bar{\nu}_l$, ...(2)

where, N is an additional nucleon required to conserve momentum of the reacting particles. MURCA is subject to the condition $p_n \leq 3p_p+p_l$. The total bulk viscosity is written as a sum of the partial bulk viscosities associated with each URCA reaction (DURCA and MURCA) [4,5,6],

$$\zeta = \zeta_{DURCA} + \zeta_{\mathrm{MURCA}}$$
$$= \sum_l \frac{|\lambda_l|}{\omega^2} \left|\frac{\partial P}{\partial Y_l}\right| \frac{\partial \eta_l}{\partial \rho} + \sum_{Nl} \frac{|\lambda_{Nl}|}{\omega^2} \left|\frac{\partial P}{\partial Y_l}\right| \frac{\partial \eta_l}{\partial \rho} \quad ....(3)$$

where ω is the frequency of the pulsation mode, $Y_l = \frac{\rho_l}{\rho}$, being the respective particle fractions, P is the pressure, $\eta_l = \mu_n - \mu_p - \mu_l$, $\rho = \rho_n + \rho_p$ is the total nucleon density, μ_i, i=n,p,e,μ, being the respective chemical potential, λ_l and λ_{Nl}determine the difference of the rates of the direct and inverse reactions of a given Urca reaction, $\left|\frac{\partial P}{\partial Y_l}\right|$ and $\frac{\partial \eta_l}{\partial \rho}$ depend on the EOS. The detail calculation is given in ref. [4,5]. The approximation of ζ is given by [see the references of 4 for details]

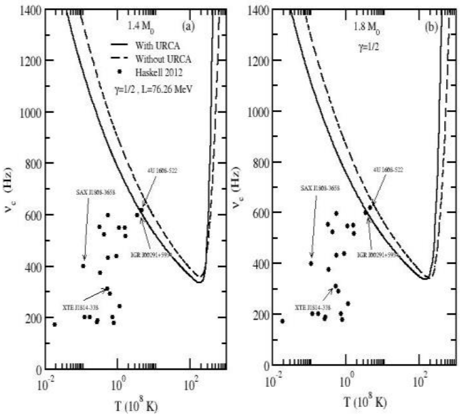
$$\zeta = 6 \text{ x } 10^{-59} \left(\frac{l+1}{2}\right)^2 \left(\frac{Hz}{\Omega}\right)^2 \left(\frac{\rho}{g\ cm^{-3}}\right)^2 \left(\frac{T}{K}\right)^6 \quad ...(4)$$

The bulk viscosity has been calculated here for the EOS of SEI of slope parameter L (ρ_0)= 76.26 MeV [7,8].

Results and Discussion

The instability boundary i.e. critical frequency (ν_c) as a function of T can is calculated obtained from the condition $\frac{1}{\tau}$ = 0 using equation 3 for taking the contribution of URCA reaction into ζ and is plotted for 1.4M⊙ and 1.8M⊙ NS in panel (a) and (b) of Figure 1 respectively. Then it is compared with the ν_c which is calculated taking equation (4) into account for 1.4M⊙ and 1.8M⊙ NS. NSs shown in figure 1 are taken from Ref. [9]. It is found that the critical frequency decreases for temperature lower than 2.7 x10^{10} K for 1.4 M⊙ and 1.8 x10^{10} K for 1.8 M⊙ and increases for temperature higher than 2.7 x10^{10} K for 1.4 M⊙ and 1.8 x10^{10} K for 1.8 M⊙ for the bulk viscosity calculated from URCA reactions compared to the critical frequency calculated for general approximation of bulk viscosity. In the latter case all the NSs [9] are found to be in stable region while 4U 1608-522 is in unstable region for ν_c calculated with URCA reactions,. It is also found that ν_c decreases in temperature lower than 2.7 x10^{10} K for 1.4 M⊙ and 1.8 x10^{10} K for 1.8 M⊙ and increases for temperature higher than 2.7 x10^{10} K for 1.4 M⊙ and 1.8 x10^{10} K for 1.8 M⊙ with increase in mass of the NS which can be seen in the figure 1.

Figure:1: The *r*-mode instability region for 1.4 and 1.8 M⊙ NS is shown for L = 76.26 MeV with the EOS of SEI.



Conclusion

A large increase in the ζ takes place as the URCA process become operative and it becomes the dominating damping mechanism to the perturbation in NSs. As a result of which the critical frequency decreases in low temperature (T< 10^{10} K) and increases in higher temperature (T >> 10^{10} K) with comparison to the critical frequency calculated from approximate expression of ζ . With increases in mass of the NS the critical frequency is lowered for low temperature and increases in high temperature.

References

[1] Andersson, N. 1998, ApJ, 502, 708

[2] Friedman, J. L., & Morsink, S. M. 1998, ApJ, 502, 714

[3] Pattnaik S P, Routray T R, Viñas X, Basu D N, Centelles M, Madhuri K and Behera B 2018 J. Phys. G: Nucl. Part. Phys. 45 055202

[4] Haensel P, Levenfish K P, and Yakovlev D G,2000, Astron. Astrophys. **357**, 1157.

[5] Haensel P, Levenfish K P, and Yakovlev D G,2000, Astron. Astrophys. **130**, 372.

[6] Vidaña Isaac,2012, Phys. Rev. C, **85** 045808.

[7] Behera B, Routray T R and Satpathy R K, 1998, J. Phys. G: Nucl. Part. Phys. **24** 2073.

[8] Behera *et. al.*,2013, J. Phys. G: Nucl. Part. Phys. **40** 095105.

[9] Haskell, B., Degenaar, N., & Ho, W. C. G. 2012, MNRAS, 424, 93.