Leptonic decays of the charged pseudoscalar mesons using Instanton liquid model

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1. Introduction

Many world wide experimental facilities like SLAC, PEP-II, BaBAR, BELLE, CLEO, ARGUS provide rich data on the charged open falvoured mesons. The leptonic decays of these open flavour charged mesons are really a challange to the experimentalists. For example, in the case of $B^+ \to \tau^+\nu_\tau$, the τ lifetime is very short so it decays very rapidly producing additional neutrinos. As these neutrinos belongs to the neutral lepton class they interacts via weak interactions only. To supplement the experimental effort theoretical estimations of such decay processes become relevant. In the present paper, we present our theoretically calculated branching ratio for the leptonic decays of the charged meson systems such as D_S^+, D^+, B^+ and K^+.

2. Theoretical Formalism

The potential we have used here is the instanton induced potential between quark and anti-quark based on Instanton liquid model [1]. The two important parameters, the average size of instanton $(\bar{\rho})$ and the average distance between instantons $(\bar{R})$ have a great consequence while describing the diluteness of the instanton medium. Numerical values of these two parameters vary for the different studies. For the present scheme we have used 0.36 fm for $(\bar{\rho})$ and 0.89 fm for $(\bar{R})$. The central part of the of the potential can be obtained by averaging the Wilson loop within the instanton ensemble [1].

For $r \ll \bar{\rho}$ i.e., when the distance between quark-antiquark is smaller than the average size of instanton, the central potential is given by [2]

$$V(r) \simeq \frac{4\pi\bar{\rho}^3}{\bar{R}^4 N_c}\left(1.345\frac{r^2}{\bar{\rho}^3} - 0.501\frac{r^4}{\bar{\rho}^4}\right) \quad (1)$$

here, $N_c = 3$ represents the colour degrees of freedom.

For $r \gg \bar{\rho}$ i.e., the distance between quark-antiquark is higher than the size of instanton, the central part of the potential is given by [2]

$$V(r) \simeq 2\Delta M_Q - \frac{g_{np}}{r} \quad (2)$$

here, $\Delta M_Q = 104.54 MeV$ is the correction to heavy-quark mass from the instanton vacuum [2]. The coupling constant $g_{NP} = 2\pi^3\bar{\rho}^4/N_c\bar{R}^4$ can be considered as nonperturbative correction to the strong coupling constant $\alpha_s(r)$. When r tends to infinity, the potential is saturated at $2\Delta M_Q$ and this suggests that instanton vacuum cannot explain quark confinement [1]. Thus, for the present study, we have added additionally a constant confinement potential. The Schrödinger equation is solved by the variational method to compute the mass spectra of the open flavour mesons. The harmonic oscillator trial wave function given by

$$R_{nl}(r) = N_{nl}(\alpha r)^l \exp\left(-\alpha^2 r^2/2\right) \text{Ł}_{n-1}^{l+1/2}\left(\alpha^2 r^2\right) \quad (3)$$

with the normalization constant expressed as

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$$N_{nl} = \sqrt{\frac{2\alpha^3\,(n-1)!}{\Gamma\,(n+l+1/2)}} \tag{4}$$

is employed for the present study. α is the variationl parameter and by minimization process of the expectation value of the hamiltonian one can determine its numeric value and also ground state energy of the system of interest. The solution of Schrödinger equation has noteworthy and decisive dependence upon the choice of the potential which acts between the constituent quark and anti-quark inside the meson.

Charged pseudoscalar mesons can decay into charged lepton pair via annihilation of the virtual W boson. As final state has highly energetic leptons and absence of hadrons, this type of decays exhibits clear experimental signatures. The decay width for this transition can be given as [3]

$$\Gamma(P \to l\nu) = \frac{G_F^2}{8\pi} f_P^2 |V_{q1q2}|^2 {m_l}^2 M_P \left(1 - \frac{m_l^2}{M_P^2}\right)^2 \tag{5}$$

where, G_F is the Fermi coupling constant, f_P represents the decay constant of the decaying pseudoscalar state. M_P stands for the pseudosclar ground state mass and m_l is serving as a leptonic mass. $V_{q_1q_2}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the constituent quarks $(q_1\bar{q}_2)$ of the parent pseudoscalar state. The decay constant which is more precisely acts as the overlap integral.

The branching ratio evaluated as

$$\mathcal{BR} = \Gamma \times \tau \tag{6}$$

Here, τ accounts for the life time of the charged pseudoscalar meson.

3. Results and Discussion

Table I comprises the our predicted branching ratio for the leptonic decays of various open flavour mesons. We have compared our predictions with the PDG [4] listed values.

TABLE I: Branching ratio for the leptonic decays of the charged pseudoscalar mesons

Channel	$\mathcal{BR}$	PDG $\mathcal{BR}$ [4]
$D_s^+ \to e^+\nu_e$	1.3378×10^{-7}	$< 8.3 \times 10^{-5}$
$D_s^+ \to \mu^+\nu_\mu$	5.6868×10^{-3}	$(5.50 \pm 0.23) \times 10^{-3}$
$D_s^+ \to \tau^+\nu_\tau$	$(5.5426)\,\%$	$(5.48 \pm 0.23)\,\%$
$D^+ \to e^+\nu_e$	8.8577×10^{-9}	$< 8.8 \times 10^{-6}$
$D^+ \to \mu^+\nu_\mu$	3.7628×10^{-4}	$(3.74 \pm 0.17) \times 10^{-4}$
$D^+ \to \tau^+\nu_\tau$	1.00348×10^{-3}	$< 1.2 \times 10^{-3}$
$B^+ \to e^+\nu_e$	1.1039×10^{-11}	$< 9.8 \times 10^{-7}$
$B^+ \to \mu^+\nu_\mu$	4.7160×10^{-7}	$(2.90 \times 10^{-7} - 1.7 \times 10^{-6})$
$B^+ \to \tau^+\nu_\tau$	1.0495×10^{-4}	$(1.09 \pm 0.24) \times 10^{-4}$
$K^+ \to e^+\nu_e$	1.6165×10^{-5}	$(1.582 \pm 0.007) \times 10^{-5}$
$K^+ \to \mu^+\nu_\mu$	$(62.9270)\,\%$	$(63.56 \pm 0.11)\,\%$

We can say that the instanton induced potential is very useful for the exploration of the leptonic decays of the charged pseudoscalar states. Our result are in an excellent agreement with the PDG data. One can compute the radiative corrections to this decay. We expect that for the case of τ because of its heavy mass radiative correction will be negligible. Also,the theoretical calculation of the decay constant and experimental measurements can collectively able to determine the CKM matrix element within standard model. Opposite to that by adopting the CKM matrix elements from unitarity, one can find the experimental values of the decay constant. This would be the suitable test for the future studies.

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