Spectroscopy of Δ Baryon

Chandni Menapara* and Ajay Kumar Rai
Department of Applied Physics, Sardar Vallabhbhai National Institute of Technology, Surat-395007, Gujarat, INDIA

Introduction

Hadron spectroscopy is a tool to reveal the wealth of strong force realm which is responsible to bind the quarks whose mediators are gluons through fundamental theory, Quantum Chromodynamics (QCD). The various experiments at CERN, Fermilab, etc. produce and detect a large number of hadrons which are very short lived through hadronization [1]. The spectroscopic study deals with hadron properties like resonance masses, magnetic moment, decay, etc. which are analyzed using various potentials in the quark model.

N and Δ are baryons with three u and d quark combinations with different isospin states. The present study encompasses Δ baryon with J=$\frac{3}{2}$ holding a place in baryon decuplet with a number of resonance states established experimentally. Δ with isospin $I = \frac{3}{2}$ has four possible states:

$$\Delta^{++}\ (I = +\tfrac{3}{2})$$
$$\Delta^{+}\ (I = +\tfrac{1}{2})$$
$$\Delta^{0}\ (I = -\tfrac{1}{2})$$
$$\Delta^{-}\ (I = -\tfrac{3}{2})$$

The foundation of QCD is based on two important properties of quarks - Confinement and Asymptotic freedom. The complete wave function for baryons being anti-symmetric includes space, spin, flavour and most importantly colour charge parts [2]. The classification of hadrons into various multiplet group as proposed by Gell-Mann owes to underlying symmetries represented by quantum numbers as electric charge, baryon number, lepton number, spin, isospin, strangeness, hypercharge and parity. The spin-flavour multiplet for SU(3) can be given as

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

There have been numerous approaches developed over time namely, Isgur-Karl Model [3], Chiral Quark Model, Lattice QCD, Quark-Diquark model [4] and so on with their own advantages and limitations. However, The phenomenological approach here is based on non-relativistic treatment of three quark dynamics using Hypercentral Constituent Quark Model [5] which overlooks gluonic interaction. Few semi-relativistic approach [6], relativistic [7] and other mass formulae have also been developed.

Theoretical Footings

An effective way to study three body systems is through consideration of Jacobi coordinates as

$$\rho = \frac{1}{\sqrt{2}}(\mathbf{r_1} - \mathbf{r_2}); \;\; \lambda = \frac{1}{\sqrt{6}}(\mathbf{r_1} + \mathbf{r_2} - 2\mathbf{r_3})$$

$$x = \sqrt{\rho^2 + \lambda^2}; \;\; \xi = arctan(\frac{\rho}{\lambda})$$

where x is hyperradius and ξ is hyperangle. Now, we incoporate the potential which solely depends on hyperradius of the system and not on hyperangle [8].

$$V(\rho, \lambda) = V(x) = -\frac{\tau}{x} + \alpha x^{\nu} + V^1(x)$$

As it is evident from the above term, the potential consists of a coulomb term and a confinement term where we have introduced

*Electronic address: `chandni.menapara@gmail.com`

a power index ν. So, we have employed hypercentral Coulomb plus Power Potential ($hCPP_\nu$). We have introduced first order correction in our potential as $\frac{1}{m}$ dependence which is of the form [9]

$$V^1(x) = -\frac{{\alpha_s}^2}{mx^2}$$

where α_s is the strong running coupling constant. Now, the complete Hamiltonian can be written as

$$H_{hCQM} = 3m + \frac{\mathbf{P_\rho^2}}{2m} + \frac{\mathbf{P_\lambda^2}}{2m} - \frac{\tau}{x} + \alpha x^\nu + V^1(x)$$

where P_ρ and P_λ are momenta conjugated to Jacobi coordinates [10]. τ and α are two free parameters which are obtained by fitting the 4* and 3* resonance masses.

Results and Discussion

The Schrodinger equation is solved using Mathematica notebook [11] which is a numerical solution rather than an analytic solution. The values hence generated are then fed to calculate resonance masses of radial 2S (S = $\frac{3}{2}, J^P = \frac{3}{2}^+$) and orbital 1P (S = $\frac{3}{2}, J^P = \frac{1}{2}^-$) excited state of Δ baryon.

In this paper, we vary the power dependence (ν) in the confinement term [12] and study its effects on the resonance masses.

State	$Mass_{exp}$ (MeV)	Power index ν	$Mass_{calc}$ (MeV)
2S	1600	0.5	1541
		1.0	1611
		1.5	1671
		2.0	1721
1P	1620	0.5	1565
		1.0	1625
		1.5	1727
		2.0	1936

It is observed that results are very close when $\nu = 1.0$ which shows the Linear dependence of the confinement potential is favoured [13]. Hence, these results can be extended for higher radial and orbital excited states with Linear confining potential.

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