Calculations of Mass Splittings in B and B_s meson systems in NRQM with Hulthen Potential*

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Introduction

The full $Q\bar{Q}$ potential used in the present work consists of a Hulthen potential and a confining linear potential. The four parameters in our model are the mass of bottom quark m_b,the mass of up quark m_u, the harmonic oscillator size parameters b and α_s. We obtain 'b' by minimizing the expectation value of the Hamiltonian There are several papers in literature where the size parameter b is defined.

The mass splitting of states with the same L but different J^{PC} values is provided by the spin-dependent interaction terms, which are added separately to the Hamiltonian. The spin-dependent parts (hyperfine and fine-structure terms) of the Hamiltonian provides the splitting of levels in B and B_s mesons, and are sensitive to the Lorentz structure of the interquark potential. Thus one has to consider the effective potential $V(r)$ as the sum of Lorentz vector (V_v) and Lorentz scalar (V_s) contributions.

splitting of the ground state which is given by

$$M(^3S_1) - M(^1S_0) = \frac{32\pi\alpha_s|\psi(0)|^2}{9m_b^2} \qquad (1)$$

Hyperfine and fine splittings

The calculations of hyperfine splitting for S-wave has been done by using the formula, $\Delta_{hf}M(nS) = M(n^3S_1) - M(n^1S_0)$; meanwhile spin-orbit splitting for P-wave states are computed by: $\Delta M(nP) = M(n^3P_2) - M(n^3P_1)$ and $\Delta M(nP) = M(n^3P_1) - M(n^3P_0)$. The hyperfine mass splitting calculated in our model is in good agreement with both experimental data collected from PDG and some of well established theoretical models. The calculated center of mass in our present work and other results of various models for B and B_s mesons is compared and listed in respective tables.

TABLE I: **Comparison of center of mass in B meson in MeV**

M_{CW}	Present	Exp.	[1]	[2]	[3]	[4]	[5]
$1\bar{S}$	5313	5313.8	5313	5315	5314	5313	5313
$2\bar{S}$	5927		5818	5902	5942	5912	5842
$3\bar{S}$	6379		6250	6385	6394	6340	6131
$4\bar{S}$	6709		6647	6785	6788		6347
$1^3\bar{P}_J$	5785	5732	5736	5736	5770	5708	5695
$1\bar{P}$	5783		5735	5745	5774	5717	5696

TABLE II: **Comparison of center of mass in B_s meson in MeV**

M_{CW}	Present	Exp.	[1]	[2]	[3]	[4]	[5]
$1\bar{S}$	5402	5313.8	5403	5404	5402	5409	5404
$2\bar{S}$	5996		5951	5988	6012	6011	5959
$3\bar{S}$	6427		6425	6473	6447	6442	6269
$4\bar{S}$	6751		6863	6878	6817		6500
$1^3\bar{P}_J$	5864		5833	5837	5850	5813	5805
$1\bar{P}$	5861		5838	5844	5852	5820	5805

Spin-dependent splittings

The spin averaged masses are defined by,

$$M(n\bar{S}) = \frac{3M(n^3S_1) + M(n^1S_0)}{4} \qquad (2)$$

$$M(n\bar{P}) = \frac{3M(n^1P_1) + 5M(n^3P_2) + 3M(n^3P_1) + M(n^3P_0)}{12} \qquad (3)$$

with n=1,2,3,... the radial quantum numbers.

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TABLE III: **Mass splitting of B meson in MeV.**

Splitting	Our work	Exp.
$1^3S_1 - 1^1S_0$	44	46
$2^3S_1 - 2^1S_0$	44	...
$3^3S_1 - 3^1S_0$	61	...
$4^3S_1 - 4^1S_0$	75	...

TABLE IV: **Mass splitting of B_s meson in MeV.**

Splitting	This work	Experiment
$1^3S_1 - 1^1S_0$	49	49
$2^3S_1 - 2^1S_0$	25	...
$3^3S_1 - 3^1S_0$	20	...
$4^3S_1 - 4^1S_0$	117	...

The corresponding spin averaged mass and spin dependent splittings are estimated and is given in the table VI.

The correctness of theoretical description of the fine splitting structure in mesons can be verified by considering the hyperfine splitting in P-state. It has been found that the measured masses of 1^1P_1 and 2^1P_1 states of mesons practically coincide with the masses of spin-averaged triplet P-state:

$$\langle M(n^3P_J)\rangle = \frac{5M(1^3P_2) + 3M(1^3P_1) + M(1^3P_0)}{9} \quad (4)$$

Practically this is the measure of centroid of the three spin-orbit split states, $^3P_0$, $^3P_1$ and $^3P_2$. The correctness is computed by considering $\Delta_{hf}(\langle M(1^3P_J)\rangle - M(n^1P_1))$, where, spin-averaged triplet P-state is estimated to be $\langle M(1^3P_J)\rangle = 5864 MeV$.

TABLE V: **The Spin-averaged and -dependent splittings for B mesons**

	States	Present	Experiment
Spin-averaged splittings	$\Delta M(1^1P_1 - 1\bar{S})$	463	....
	$\Delta M(1^3P_0 - 1\bar{S})$	404	397
	$\Delta M(1^3P_1 - 1\bar{S})$	471	410
	$\Delta M(1^3P_2 - 1\bar{S})$	483	430

TABLE VI: **The Spin-averaged and -dependent splittings for B_s mesons**

	States	Present	Experiment
Spin-averaged splittings	$\Delta M(1^1P_1 - 1\bar{S})$	540	516
	$\Delta M(1^3P_0 - 1\bar{S})$	514	....
	$\Delta M(1^3P_1 - 1\bar{S})$	545	540
	$\Delta M(1^3P_2 - 1\bar{S})$	563	527

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