Charge and thermal transport properties of a hot QCD matter in a strong magnetic field

Shubhalaxmi Rath* and Binoy Krishna Patra†
Department of Physics, Indian Institute of Technology Roorkee, Roorkee 247667, India

Introduction

Ultrarelativistic heavy-ion collisions at RHIC and LHC confirm that a normal nuclear matter undergoes into a hot quark matter, dubbed as quark-gluon plasma (QGP). In addition, the collision for nonzero impact parameter also produces a strong magnetic field, whose magnitude varies from $eB = m_\pi^2$ ($\simeq 10^{18}$ Gauss) at RHIC to 15 m_π^2 at LHC. The time-span for which these magnetic fields remain strong, depends mainly on the charge transport properties of the medium produced, and the degree of local equilibrium of the aforesaid medium can be answered by its thermal transport properties. These facts motivate to calculate the electrical ($\sigma_{\rm el}$) and thermal (κ) conductivities in the absence and presence of strong magnetic field to isolate the effects of the magnetic field alone. Once we understand these transport properties, we can further study the effect of strong magnetic field on the relative behavior between them by the Lorenz number in Wiedemann-Franz law and the test of equilibration by the Knudsen number (Ω) through κ, with respect to its counterpart at $B = 0$.

In the presence of a strong magnetic field ($|q_iB| \gg T^2$ and $|q_iB| \gg m_i^2$), the motion of particle along the longitudinal direction (p_L) (along the direction of magnetic field) is much greater than the motion along the transverse direction (p_T), *i.e.* $p_L \gg p_T$, so an anisotropy is resulted in the momentum space. Thus the anisotropic parameter, ξ $((\langle p_T^2\rangle/(2\langle p_L^2\rangle)) - 1)$ is negative, and it describes the stretching of the distribution along the direction of anisotropy.

We have earlier observed the effects of the strong magnetic field on the thermodynamic and magnetic properties of QCD medium [1, 2]. In this paper, we intend to revisit the charge and thermal transport properties and the relative behavior between them through the Lorenz number and the Knudsen number in the strong magnetic field-driven anisotropy. We also wish to observe how these properties are different from their respective behaviors in the isotropic medium. To calculate the electrical and thermal conductivities, we have used the kinetic theory approach by solving the relativistic Boltzmann transport equation in the relaxation-time approximation, where the interactions among partons are incorporated through their effective thermal masses.

Charge and thermal transport properties

The electrical and thermal conductivities for an isotropic medium are given by

$$\sigma_{\rm el}^{\rm iso} = \frac{2\beta}{3\pi^2}\sum_i g_i q_i^2 \int d{\rm p}\frac{{\rm p}^4}{\omega_i^2}\tau_i f_i^{\rm iso}(1-f_i^{\rm iso}) \ , \quad (1)$$

$$\kappa^{\rm iso} = \frac{\beta^2}{3\pi^2}\sum_i g_i \int d{\rm p}\frac{{\rm p}^4}{\omega_i^2}(\omega_i - h_i)^2\tau_i \times f_i^{\rm iso}(1-f_i^{\rm iso}) \ , \quad (2)$$

respectively. In the presence of a strong magnetic field, these conductivities are calculated [3] as

$$\begin{aligned}\sigma_{\rm el,B}^{\rm aniso} &= \frac{\beta}{\pi^2}\sum_i g_i q_i^2|q_iB|\int dp_3\frac{p_3^2}{\omega_i^2}\tau_i^B f_i(1-f_i) \\ &- \xi\left[\frac{\beta^2}{2\pi^2}\sum_i g_i q_i^2|q_iB|\int dp_3\frac{p_3^4}{\omega_i^3}\tau_i^B f_i \right.\\ &\times (1-f_i)\left\{1-2f_i+\frac{1}{\beta\omega_i}\right\} - \frac{\beta}{2\pi^2} \\ &\times \left.\sum_i g_i q_i^2|q_iB|\int dp_3\frac{p_3^2}{\omega_i^2}\tau_i^B f_i(1-f_i)\right] , \quad (3)\end{aligned}$$

$$\begin{aligned}\kappa_{\rm B}^{\rm aniso} &= \frac{\beta^2}{2\pi^2}\sum_i g_i|q_iB|\int dp_3\frac{p_3^2}{\omega_i^2}(\omega_i - h_i^B)^2\tau_i^B f_i \\ &\times (1-f_i) + \xi\left[\frac{\beta^2}{4\pi^2}\sum_i g_i|q_iB|\int dp_3\frac{p_3^4}{\omega_i^4}\right. \\ &\times (\omega_i^2 - h_i^{B\,2})\tau_i^B f_i(1-f_i) \\ &- \frac{\beta^3}{4\pi^2}\sum_i g_i|q_iB|\int dp_3\frac{p_3^4}{\omega_i^3}(\omega_i - h_i^B)^2\tau_i^B \\ &\times \left. f_i(1-2f_i)(1-f_i)\right] . \quad (4)\end{aligned}$$

---

*Electronic address: srath@ph.iitr.ac.in
†Electronic address: binoyfph@iitr.ac.in

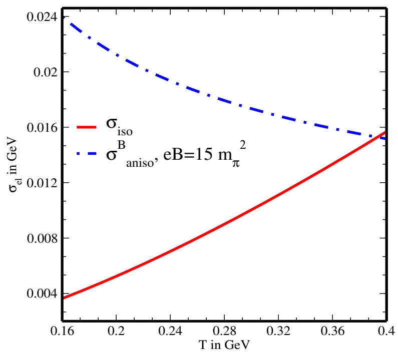

FIG. 1: Variations of $\sigma_{\rm el}$ with temperature in isotropic and magnetic field-driven anisotropic mediums.

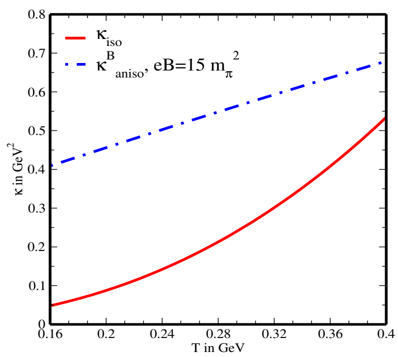

FIG. 2: Variations of κ with temperature in isotropic and magnetic field-driven anisotropic mediums.

In the calculation, we have set the magnetic field at 15 m_π^2 and the weak-momentum anisotropic parameter (ξ) at -0.6. We have used the thermal mass (squared) of quark, which has value $m_i^2 = g^2T^2/6$ at $T \neq 0$ and $m_i^2 = \frac{g^2|q_iB|}{3\pi^2}\left[\frac{\pi T}{2m_i} - \ln(2)\right]$ at $T \neq 0$ and $B \neq 0$, for ith flavor.

From figures 1 and 2, we have noticed that both electrical and thermal conductivities get increased in a magnetic field-driven anisotropy. Here $\sigma_{\rm el}$ is seen to decrease with temperature, opposite to its increasing behavior in the isotropic medium. Whereas κ shows slow increasing trend with temperature, contrary to its rapid increasing trend in the isotropic medium. These differences are mainly due to three factors: the stretching of distribution function due to magnetic field-driven anisotropy, the relaxation time in the absence and presence of magnetic field and the dispersion relation. These behaviors of two conductivities facilitate to understand the Wiedemann-Franz law and the local equilibrium of the medium.

The Wiedemann-Franz law helps to understand the relation between the charge transport and the thermal transport in a system. This law states that the ratio of charged particle contribution of the thermal conductivity to the electrical conductivity is equal to the product of the Lorenz number (L) and the temperature,

$$\frac{\kappa}{\sigma_{\rm el}} = LT \ . \tag{5}$$

The metals which are good thermal and electrical conductors, perfectly satisfy this law. So $\kappa/\sigma_{\rm el}$ has approximately the same value for different metals at the same temperature, *i.e.* Lorenz number $(\kappa/(\sigma_{\rm el}T))$ remains the same. For hot QCD matter, we have observed that, the ratio $\kappa/\sigma_{\rm el}$ rises linearly with the temperature in the presence of magnetic field-driven anisotropy, with a magnitude larger than that in the isotropic medium. So, at a fixed temperature, the Lorenz number in the magnetic field-driven anisotropy is larger than that in the isotropic medium. Thus the dominance of thermal conductivity over electrical conductivity gets increased in the strong magnetic field regime.

The local equilibrium of a medium can be understood through the Knudsen number,

$$\Omega = \frac{\lambda}{l} = \frac{3\kappa}{luC_V} \ , \tag{6}$$

where λ, l, C_V and u are the mean free path, the length scale of the system, the specific heat at constant volume and the relative speed, respectively. We have found that, the presence of strong magnetic field makes the Knudsen number larger than its value in the isotropic medium, but it remains less than one. Thus the hot QCD matter may remain in local equilibrium even in the presence of weak-momentum anisotropy induced by the strong magnetic field.

References

[1] S. Rath and B. K. Patra, JHEP **1712**, 098 (2017).
[2] S. Rath and B. K. Patra, arXiv:1806.03008 [hep-th].
[3] S. Rath and B. K. Patra, Phys. Rev. D **100**, 016009 (2019).