NJL model estimation of anisotropic electrical conductivity for quark matter in presence of magnetic field

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Present work has gone through the microscopic calculation of electrical conductivity of quark matter in presence of magnetic field, where Nambu-Jona-Lasinio model is considered for mapping the interaction picture of the medium. Let us start with Ohm's law'

$$J_D^i = \sigma^{ij} E_j \quad (1)$$

where J_D^i is dissipative current, σ^{ij} electricl conductivity tensor and E_j is electric field. Now for a fluid of quark having spin-color degeneracy g and electric charge q_f dissipative current from kinetic theory framework can be written as,

$$J_D^j = q_f g \int \frac{d^3p}{(2\pi)^3} \vec{v}\, \delta f \quad (2)$$

Where δf is small deviation of quark distribution function from the equilibrium Fermi-Dirac distribution of quark $f_0 = \frac{1}{e^{\beta\omega}+1}$. Interms of 3-momentum ($\vec{p}$) and energy (ω) particle velocity can be writen as $\vec{v} = \frac{\vec{p}}{\omega}$ with $\omega = \sqrt{\vec{p}^2 + M^2}$. Now to find δf in presence of electric field $\vec{E}$ and magnetic field $\vec{B}$ we use relaxation time approximation (RTA) in Boltzmann's equation, where we can assume a general force term

$$\vec{\mathcal{F}} = \alpha\vec{e} + \beta\vec{b} + \gamma\vec{e}\times\vec{b} \quad (3)$$

where $\vec{e}$, $\vec{b}$ are unit vectors along $\vec{E}$ and $\vec{B}$. Connecting δf and $\vec{\mathcal{F}}$ suitably [1, 2], the coefficients α, β and γ can be found as

$$\begin{aligned} \alpha &= q\left(\frac{\tau_c}{\omega}\right)\frac{1}{1+(\tau_c/\tau_B)^2}\,E, \\ \beta &= q\left(\frac{\tau_c}{\omega}\right)\frac{(\tau_c/\tau_B)^2}{1+(\tau_c/\tau_B)^2}\,(\vec{e}\cdot\vec{h})E, \\ \gamma &= -q\left(\frac{\tau_c}{\omega}\right)\frac{(\tau_c/\tau_B)}{1+(\tau_c/\tau_B)^2}\,E, \end{aligned} \quad (4)$$

where τ_B and τ_c as magnetic and thermal relaxation time. After taking care of all degeneracy factors of u and d quarks, we get the 3 components of electrical conductivity, whose general expressions can be written as

$$\sigma_n = e^2\beta\,\frac{20}{9}\int\frac{d^3p}{(2\pi)^3}\frac{\vec{p}^2}{\omega^2}\frac{\tau_c(\tau_c/\tau_B)^n}{1+(\tau_c/\tau_B)^2}f_0(1-f_0). \quad (5)$$

with $n = 0, 1, 2$. We will use temperature (T) and magnetic field (B) dependent effective quark mass from NJL model, briefly discussed below, in Eq. (5) to estimate σ_n of quark matter.

The Lagrangian density for the isospin-symmetric ($m_u = m_d$) two-flavor version of NJL model in presence of electromagnetic field (A^μ) is given by

$$\begin{aligned} \mathcal{L}_{NJL} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left(\not{D} - m\right)\psi \\ &+G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right], \end{aligned} \quad (6)$$

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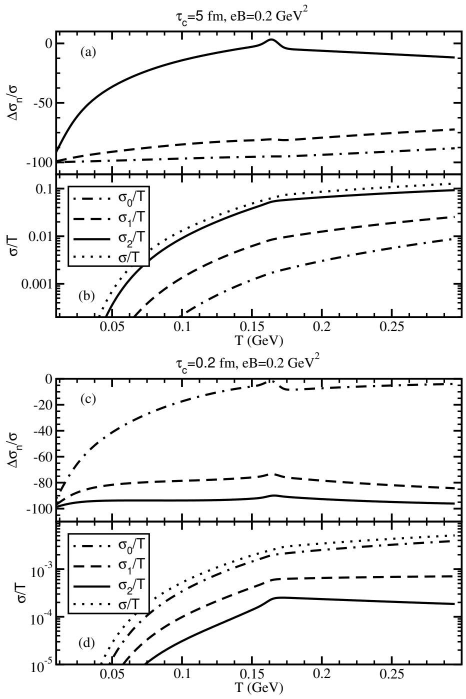

FIG. 1: Temperature dependence of three components of electrical conductivities ($\sigma_{0,1,2}$) with $eB = 0.2$ GeV^2 and σ (without B) for $\tau_c = 5$ fm (b) and 0.2 fm (d). $\Delta\sigma_n/\sigma = (\sigma_n - \sigma)/\sigma$ for $\tau_c = 5$ fm (a) and 0.2 fm (c).

Where, $D_\mu = i\partial_\mu - QA_\mu$ with $Q = diag(q_u = 2e/3, q_d = -e/3)$ as the charge matrix. In quasi-particle approximation, the gap equation for the constituent quark mass M at finite T and B is given by

$$M(B,T) = m - 2G(B,T) \sum_{f=u,d} \langle \bar{\psi}_f \psi_f \rangle, \quad (7)$$

where $\langle \bar{\psi}_f \psi_f \rangle$ represents the quark condensate of flavor f, and a thermo-magnetic NJL coupling constant $G(B,T)$ has been considered [3, 4]. In Figs. 1(b) and (d), we present the temperature dependence of the different components of electrical conductivities σ_n (scaled with T) in presence of an external magnetic field as well as the without field case,

$$\sigma = e^2 \beta \, \frac{20}{9} \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^2}{\omega^2} \tau_c f_0 (1 - f_0). \quad (8)$$

The difference $\frac{\Delta\sigma_n}{\sigma} = \frac{(\sigma_n - \sigma)}{\sigma}$ are also plotted in Figs. 1(a) and (c) which emphasize the effect of external magnetic field on each components. The results are presented with a fixed value of $eB = 0.2$ GeV^2 but for two different values of τ_c, i.e. $\tau_c = 5fm$ (a,b) $\tau_c = 0.2fm$ (c,d) , which can be assigned with the zones $\tau_c > \tau_B$ and $\tau_c < \tau_B$ respectively. It means that $eB = 0.2$ GeV^2 may be considered as stronger magnetic field for $\tau_c = 5$ fm and weaker magnetic field for $\tau_c = 0.2$ fm. Therefore, former case is showing $\sigma_2 > \sigma_0$ and latter case is showing $\sigma_2 < \sigma_0$. It is controlled by the anisotropic function $\frac{(\tau_c/\tau_B)^n}{1+(\tau_c/\tau_B)^2}$. In terms of the anisotropy, the above outcomes can be briefly sketched, or:

$$\begin{aligned} \text{for } \tau_c &= 5 \text{ fm} \\ \sigma^{xx} &= \sigma^{yy} < \sigma^{zz} \Rightarrow \text{larger anisotropy} \quad (9) \\ \text{for } \tau_c &= 0.2 \text{ fm ,} \\ \sigma^{xx} &= \sigma^{yy} \approx \sigma^{zz} \Rightarrow \text{smaller anisotropy} \quad (10) \end{aligned}$$

when external magnetic field $eB = 0.2$ GeV^2 is along the z-direction. As seen from Fig. 1, all the components of the electrical conductivities increase with temperature with a kink near the quark-hadron phase transition temperature T_c and rate of increments are also different for two different phases.

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