

## A root finding method for parametrizing the chemical freeze-out surface in heavy-ion Collision Experiments

Deeptak Biswas<sup>1,\*</sup>, Rajarshi Ray<sup>1</sup>, Sumana  
Bhattacharyya<sup>1</sup>, Sanjay K. Ghosh<sup>1</sup>, and Pracheta Singha<sup>1</sup>  
<sup>1</sup>*Department of Physics, Center for Astroparticle Physics & Space Science,  
Bose Institute, EN-80, Sector-5, Bidhan Nagar, Kolkata-700091, India*

A new prescription for obtaining the chemical freeze-out parameters in the heavy-ion collision experiments have been introduced. A root-finding process has been developed rather than the standard minimization techniques. The scheme has reliably estimated the freeze-out parameters and predicted the hadron yield ratios.

### 1. Introduction

The chemical freeze-out (CFO) surface in a heavy-ion collision is expected to provide us information about the chemical equilibration of the strongly interacting matter. The striking similarity between the CFO surface and phase transition boundary makes it an important tool for studying the QCD matter. Here we have proposed a novel procedure for the extraction of thermal parameters from experimental hadron yields. We have constructed the net baryon charge out of experimental hadron yield. This net charge has been utilized with that available from HRG model to extract information of CFO.

### Hadron Resonance Gas model

The grand canonical partition function of the hadron resonance gas is given by,

$$\ln Z^{ideal} = \sum_i \ln Z_i^{ideal}, \quad (1)$$

The sum runs over all hadrons and resonances. Logarithm of partition function for  $i$ 'th species is given as,

$$\ln Z_i^{ideal} = \pm \frac{Vg_i}{(2\pi)^3} \int d^3p \ln[1 \pm \exp(-(E_i - \mu_i)/T)] \quad (2)$$

Here, upper sign is for baryons and lower for mesons,  $V$  is the volume,  $T$  is the tempera-

ture,  $g_i$ ,  $E_i$  and  $m_i$  are respectively the degeneracy factor, energy and mass of the  $i$ 'th hadron.  $\mu_i = B_i\mu_B + Q_i\mu_Q + S_i\mu_S$  is the chemical potential, with  $B_i$ ,  $Q_i$  and  $S_i$  denoting the baryon number, electric charge and strangeness respectively. Here  $\mu_B$ ,  $\mu_Q$  and  $\mu_S$  are the baryon, electric and strangeness chemical potentials respectively. For a thermalized system the number density  $n_i$  can be calculated from partition function.

$$n_i(T, \mu_B, \mu_Q, \mu_S) = \frac{g_i}{(2\pi)^3} \int \frac{d^3p}{\exp[(E_i - \mu_i)/T] \pm 1} \quad (3)$$

### 2. Standard approach

For  $\chi^2$  fitting procedure one can construct the  $\chi^2$  out of data and model prediction as,

$$\chi^2 = \sum_i \frac{(Ratio_i^{Model} - Ratio_i^{Expt})^2}{\sigma_i^2} \quad (4)$$

In standard practice, one can construct various yield ratios to nullify volume and other systematics. Then that information is fed down in the chi-square. Then the constructed chi-square is minimized for the parameters to be fitted. Alongside these minimization equations, two extra constraint conditions are also imposed demanding the strange neutrality and net baryon to net charge ratio of the incident nuclei.

### 3. The new approach

Here, instead of the minimization equations concerning  $T$  and  $\mu_B$ , we introduce two new

---

\*Electronic address: [deeptakb@gmail.com](mailto:deeptakb@gmail.com)

independent equations. The net baryon number normalized to the total baryon number and net baryon number normalized to the total hadron yield have been proposed.

$$\frac{\sum_i^{Det} B_i \frac{dN_i}{dY}}{\sum_i^{Det} |B_i| \frac{dN_i}{dY}} = \frac{\sum_i^{Det} B_i n_i^{Tot}}{\sum_i^{Det} |B_i| n_i^{Tot}} \quad (5)$$

$$\frac{\sum_i^{Det} B_i \frac{dN_i}{dY}}{\sum_i^{Det} \frac{dN_i}{dY}} = \frac{\sum_i^{Det} B_i n_i^{Tot}}{\sum_i^{Det} n_i^{Tot}} \quad (6)$$

The ratios on the LHS of the above equations consists of the mid-rapidity data of HIC experiments and those on the right are the number densities calculated in the HRG model. The sum runs over the identified hadrons for which the yield data are available. The equations are unique and independent of each other for a sufficient number of identified hadrons.

## 4. Results

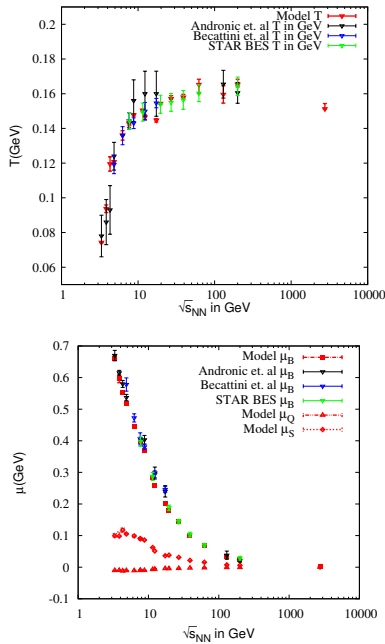


FIG. 1: Variation of  $T$ ,  $\mu_B$ ,  $\mu_Q$ ,  $\mu_S$  with  $\sqrt{s}$

The equations Eq. (5-6) along with two constraints have been solved with an accuracy of

$10^{-6}$ . The freeze-out parameters are depicted in Fig. 1. The general behavior as well as the quantitative estimates are commensurate with those in the existing literature. The variation of the freeze-out temperature with the center of mass energy  $\sqrt{s}$  is upper panel of Fig. 1. In lower panel the various chemical potentials are shown as functions of  $\sqrt{s}$ . As mentioned

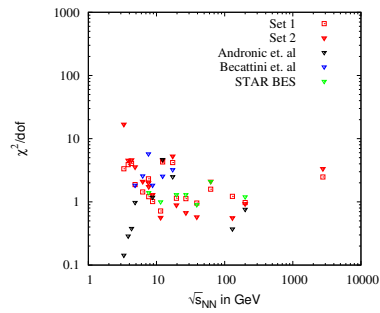


FIG. 2: Variation of  $\chi^2/dof$

above we have not done a  $\chi^2$  fit. But for comparison, we have plotted  $\chi^2$  to degrees of freedom (d.o.f) ratio from our resulted parameter set. We have used two sets for better judgment.  $\chi^2$  to d.o.f has quite good agreement with results from standard chi-square approach. The variation of  $\chi^2/dof$  with  $\sqrt{s}$  is shown in Fig. 2, along with some available results in the literature.

## Acknowledgments

We acknowledge UGC, DST and CSIR for supporting this work.

## References

- [1] S. Bhattacharyya, D. Biswas, S. K. Ghosh, R. Ray and P. Singha, Phys. Rev. D **100**, 054037 (2019) doi:10.1103/PhysRevD.100.054037 [arXiv:1904.00959 [nucl-th]].
- [2] A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nucl. Phys. A **789**, 334 (2007) doi:10.1016/j.nuclphysa.2007.02.013 [nucl-th/0611023].