A Direct Mathematical Method to Calculate the Efficiency of 4π Sum-Spin Spectrometer

A. Srivastava[1], S. Panwar[2], V. Ranga[2], and G. Anil Kumar[2*]
[1]Dept. of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee-247667, Uttarakhand, INDIA and
[2]Radiation Detectors and Spectroscopy Laboratory, Dept. of Physics, Indian Institute of Technology Roorkee, Roorkee-247667, Uttarakhand, INDIA

Introduction

The absolute detection efficiency of a radiation detector can be determined using various methods such as experimental measurements, simulations, analytical and numerical approach. Using direct mathematical method, several authors have reported the efficiency of detectors of various geometrical size and shape such as cylindrical, conical, straight hexagonal, parallelepiped, spherical, elliptical, etc. [1,2]. These efficiencies were calculated for axially and non-axially positioned gamma sources. The formulae derived so far for the calculation of absolute detection efficiency are in the double integral form and have general description of the solid angle in terms of polar angles and azimuthal angles. The integration has been performed either by using simulations or by numerical approach without providing the solution of integration in the closed form. To the best of our knowledge, no exact formula for the solid angle subtended by an n-sided polygonal cross-section is available in the literature. In addition, the formula for calculation of error associated with the absolute detection efficiency due to uncertainty in the source-detector distance has not been developed. In the present work, we have developed an exact formula for the solid angle subtended by an n-sided polygonal cross-section at axially positioned isotropic point source. The solid angle formula, so derived, has been applied to calculate the absolute total detection efficiency of NaI(Tl) scintillation detectors of pentagonal and hexagonal cross sections for mono-energetic gamma rays. These efficiency values are then used to calculate the absolute total detection efficiency of 4π sum-spin spectrometer for gamma rays of different energies. The spectrometer is a close packed array of 12 conical pentagonal and 20 conical hexagonal detectors. The primary role of this spectrometer is to measure the multiplicity and sum-energy of the discrete low energy gamma rays emitted in heavy-ion induced fusion-evaporation reactions. This spectrometer is being used in conjunction with a high energy gamma ray spectrometer for detection of phase space selected high energy GDR gamma rays. Since it's commissioning the array has been successfully used in in-beam experiments at TIFR, Mumbai and IUAC, New Delhi.

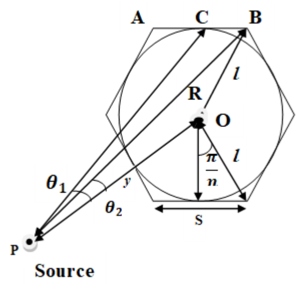

FIG. 1: Schematic diagram for an isotropic point source placed axially at a distance y from the conical detector of an n-sided polygonal cross-section of circumradius l represented here as a hexagon.

Theoretical formalism

Consider a conical n-sided polygonal detector of circumradius l of front-end polygon, and an axially located isotropic point source at the

*Electronic address: anilgfph@iitr.ac.in, anilgouri@gmail.com

distance of y from the detector (see figure-1.) The absolute total detection efficiency can be expressed as

$$\epsilon_{abs} = G \times I \times M \tag{1}$$

Here, $G = \frac{\Omega}{4\pi}$ where Ω denotes the total solid angle at the front-end polygonal surface. The measure of the fraction of photons transmitted by the intervening materials to the detector surface is $I = \Sigma_i e^{-\mu_i d_i}$ where μ_i is the attenuation coefficient of the i^{th} absorber and d_i is the distance travelled by the photon in the i^{th} absorber. M is the measure of the fraction of photons absorbed by the detector and is given by $M = 1 - e^{-\mu t}$ where μ is the attenuation coefficient of the detector for the photons and t is the average path length travelled by the photon. The technique involves the subtraction of solid angle in which photons do not enter the detector from that of the circular cross-section. For n-sided polygon, the total azimuthal angle in which photons do not enter the detector is given by $2n\alpha$ where 2α is the angle for one triangle in the polygon. The solid angle can be calculated using following equation

$$\Omega = 2\pi \times \left[1 - \frac{n}{\pi}\cos^{-1}\left[\left(\cos\frac{\pi}{n}\right)\sqrt{\frac{l^2+y^2}{l^2\cos^2\frac{\pi}{n}+y^2}}\right]\right] \tag{2}$$

For the detector of length x and circumradius of front-end polygon l,

$$\epsilon_{abs} = \frac{\Omega}{4\pi} \times \Sigma_i e^{-\mu_i d_i} \times \left[1 - \frac{1}{\Omega}\begin{pmatrix} 2\pi\int_0^{\theta_1} e^{-\mu(E)x/\cos\theta}\sin\theta d\theta + \\ 2\int_{\theta_1}^{\theta_2}(\pi - n\alpha)e^{-\mu(E)x/\cos\theta}\sin\theta d\theta \end{pmatrix}\right] \tag{3}$$

where $\theta_1 = \tan^{-1}(l\cos(\pi/n)/y)$,
$\theta_2 = \tan^{-1}(l/y)$,
$\alpha = \cos^{-1}[(l\cos(\pi/n)\cot\theta)/y]$

Considering the uncertainty in the detector-source distance, the relative error in absolute efficiency is given by

$$\left|\frac{d\epsilon_{abs}}{\epsilon_{abs}}\right| = \frac{1}{M\Omega}\left[\frac{nl^2\sin(2\pi/n)}{(l^2\cos^2\frac{\pi}{n}+y^2)\sqrt{l^2+y^2}}\right]|dy| \tag{4}$$

Results and Discussion

In order to validate the formalism developed, the efficiency values are compared with results obtained from realistic GEANT4 simulations for a wide gamma energy range. In addition, the theoretical values were compared with measured results obtained for conical hexagon, conical pentagon and 4π-array for 662 keV [3, 4] and summarized in Table I. A good agreement within the permissible errors has been found.

TABLE I: Simulated, measured and theoretical efficiencies of different shapes of NaI(Tl) detectors for 662 keV gamma rays.

Shape	Simul.	Exp.	Theory
Conical hexagon	3.05(0.06)	3.09(0.12)	2.98(0.1)
Conical pentagon	2.05(0.06)	2.06(0.12)	2.03(0.1)
4π array	78.2(1.1)	77.7(3.1)	83.1(3.3)

Acknowledgements

Special thanks to Prof. Indranil Mazumdar, Tata Institute of Fundamental Research, Mumbai for his help. One of the authors V. Ranga would like to acknowledge the support provided by CSIR-JRF fellowship under Grant No. 09/143(0907)/2017-EMR-I.

References

[1] A. Hamzawy *et al.*, Rev. Sci. Instr. 87(2016)115105
[2] M. I. Abbas *et al.*, Nucl. Instr. and Meth. 771 (2015) 121
[3] G. Anil Kumar *et al.*, Proc. IEEE Nucl. Sci. Symp. N17-1 (2008) 1640
[4] G. Anil Kumar *et al.*, Nucl. Instr. Meth. 611 (2008) 76.