Equations of States for White Dwarfs and Neutron Stars

Somnath Mukhopadhyay*
Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata - 700064, INDIA and Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai - 400085, INDIA†

Compact stars are exotic astrophysical objects formed as stellar remnants when a normal star runs out of nuclear fuel. Based on the nature of the remnant, they are categorized into three classes, namely, white dwarfs, neutron stars and black holes. A star shines steadily due to balance between the gravitational pressure and pressure due to nuclear fusion and thus loses its nuclear energy reservoir in a finite amount of time. When a star has exhausted all of its energy, the gas pressure of the hot interior can no longer support the weight of the star and the star collapses into a denser state and thus a compact star is born. The formation of the various kinds of compact stars depends on the mass of the progenitor star prior to the gravitational collapse. Stars with masses less than $\sim 10M_\odot$ at the end of their lifetime collapse into white dwarfs, those with $\sim 10 - 20M_\odot$ form neutron stars and those with masses greater than $20M_\odot$ form black holes. A white dwarf is a compact star whose mean mass is of the order of a Solar mass and mean radius of the order of 1000 kms. Hence its mean density is $\sim 10^9$ gms/cc, making a white dwarf one of the densest objects in the Universe. A white dwarf's interior is degenerate, and it is supported against gravitational collapse by the pressure of the extremely relativistic free electron gas. Since electrons are fermions and they are degenerate inside a white dwarf, at such densities they have extremely large kinetic energies and hence large pressure which is enough to support them against gravity. The atomic nuclei contributes mainly to the gravitational mass of a white dwarf. A neutron star's mass is of the order of one Solar mass but in contrast to a white dwarf, its radius is of the order of only 10 kms. Hence the mean density of a neutron star is $\sim 10^{15}$ gms/cc, making a neutron star's interior to be the densest matter in the Cosmos. A neutron star is born from the violent core-collapse supernova explosion of a massive star generally known as Type II supernova. A neutron star's interior is enriched with a plethora of exotic phases of dense matter which makes its composition or the Equation of State (EoS) a total mystery till today.

Recently, several white dwarfs have been proposed with masses significantly above the Chandrasekhar limit, known as Super-Chandrasekhar White Dwarfs, to account for the overluminous Type Ia supernovae. In the first part of the thesis, the EoS of a completely degenerate relativistic electron gas in magnetic field based on Landau quantization of charged particles in a magnetic field is developed. The presence of Coulomb screening in presence of magnetic field within the relativistic Thomas-Fermi model is also investigated. The mass-radius relations for magnetized White Dwarfs are obtained by solving the Tolman-Oppenheimer-Volkoff equations. The effects of the magnetic energy density and pressure contributed by a density-dependent magnetic field are treated properly to find the stability configurations of realistic magnetized super-Chandrasekhar white dwarf stars [1, 2].

In the second part of the thesis, the core-crust transition and crustal fraction of moment of inertia in neutron stars is calculated using β-equilibrated nuclear matter obtained from density dependent M3Y effective interaction (DDM3Y). The transition density, pressure and proton fraction at the inner edge sep-

*Electronic address: smpapan7@gmail.com;somnathm@hyderabad.bits-pilani.ac.in
†Also at BITS Pilani Hyderabad Campus, Hyderabad, Telangana - 500 078, INDIA

arating the liquid core from the solid crust of the neutron stars determined from the thermodynamic stability conditions are found to be $\rho_t = 0.0938 fm^{-3}$, $P_t = 0.5006 MeV fm^{-3}$ and $x_{p(t)} = 0.0308$, respectively. The crustal fraction of the moment of inertia can be extracted from studying pulsar glitches and is most sensitive to the pressure as well as density at the transition from the crust to the core. These results for pressure and density at core-crust transition together with the observed minimum crustal fraction of the total moment of inertia provide a new limit for the radius of the Vela pulsar: $R \geq 4.10 + 3.36 M/M_\odot$ kms [3].

In the third part, the Rossby mode (r-mode) instability windows, spindown, spindown rates and the gravitational wave signatures of neutron stars in the slow rotation approximation using the equation of state obtained from the density dependent M3Y effective interaction are studied. The neutron star matter is taken to be β-equilibrated neutron-proton-electron matter at the core with a rigid crust. The fiducial gravitational and viscous timescales, the critical frequencies and the time evolutions of the frequencies and the rates of frequency change are calculated for a range of neutron star masses. It is shown that the young and hot rotating neutron stars lie in the r-mode instability region. It is also emphasized that if the dominant dissipative mechanism of the r-mode is the shear viscosity along the boundary layer of the crust-core interface, then the neutron stars with low value of slope of symmetry energy L lie in the r-mode instability region and hence emit gravitational radiation [4]. The effects of bulk viscosity and the shear viscosity in the core, using r-mode amplitude constraints from observed luminosities of neutron stars has recently also been explored [5]. In both the above scenarios it is found that none of the observed Millisecond Radio Pulsars (MSRPs) and pulsars in Low Mass X-ray Binaries (LMXBs) lie in the r-mode instability region and hence do not emit gravitational waves.

In the fourth part, the masses and radii of non-rotating and rotating configurations of pure hadronic stars mixed with self-interacting fermionic asymmetric dark matter are calculated within the two-fluid formalism of stellar structure equations in general relativity. The EoS of nuclear matter is obtained from the density dependent M3Y effective nucleonnucleon interaction. The dark matter particle mass is taken to be of 1 GeV. The EoS of self-interacting dark matter is taken from two-body repulsive interactions of the scale of strong interactions within an effective vector field theory. The dark matter mediator mass is taken to be 100 MeV. Such strongly self-interacting dark matter is consistent with the observed distributions of dark matter in galaxies and also with the number of satellite galaxies of a primary galaxy. The conditions of equal and different rotational frequencies of nuclear matter and dark matter in a dark matter admixed neutron star are explored considering the interaction between nuclear (baryonic) matter and dark matter purely gravitational. It is found that the maximum mass of strongly self-interacting fermionic asymmetric dark matter dominated differentially rotating neutron stars is $\sim 1.94 M_\odot$ with radius ~ 10.4 kms [6].

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