Proton radius of ^{13}C using *ab initio* no-core shell-model

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Introduction

Advances in the development of *ab initio* nuclear many-body methods have opened a new path in the understanding of open shell nuclei. No-core shell-model (NCSM) [1–4] is one such microscopic *ab initio* approach in which realistic interaction is used to explain the low-lying energy spectra and other observables. One key challenge in nuclear theory is to construct these realistic interactions occuring between protons and neutrons. These fundamental interactions are derived from first principles. In NCSM, all nucleons are treated as active and thus, the concept of inert core does not exist, unlike in the conventional shell-model. New experiments have revealed some exotic properties in carbon chain like subshell closure at $Z = 6$, which is not explained by shell-model with phenomenological interaction thereby necessitating *ab initio* calculations. We have carried out a comprehensive study of carbon chain using NCSM with realistic nucleon-nucleon (NN) interactions. In this paper, we have presented results for ^{13}C.

Formalism

We consider a system which consists of A interacting nucleons. The Hamiltonian for such a system in NCSM method can be expressed as

$$H_A = \frac{1}{A}\sum_{i<j}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{ij}^{NN}$$

In the above expression, first term is the relative kinetic energy and the second term represents the two-body interaction, where, m is the nucleon mass.

We add harmonic oscillator (HO) centre-of-mass (CM) Hamiltonian to the main Hamiltonian thereby making the latter frequency dependent.

$$H_A^{\Omega} = H_A + H_{CM} = \sum_{i=1}^{A} h_i + \sum_{i<j}^{A} V_{ij}^{\Omega,A}$$
$$= \sum_{i=1}^{A}\left[\frac{\vec{p}_i{}^2}{2m} + \frac{1}{2}m\Omega^2\vec{r}_i{}^2\right] + \sum_{i<j}^{A}\left[V_{ij}^{NN} - \frac{m\Omega^2(\vec{r}_i - \vec{r}_j)^2}{A}\right]$$

Here, $H_{CM} = T_{CM} + U_{CM}$. The intrinsic properties of the system are not affected by the HO CM Hamiltonian on account of the translational invariance of Hamiltonian H_A. We subtract this H_{CM} in the final NCSM calculation.

In the NCSM method, HO basis states are employed in truncated space ($N_{\max}$), where, $N_{\max}$ is defined as the number of HO excitations above the minimum A-nucleon configuration. The realistic NN potentials act in the complete Hilbert space. As a result of the truncation of the HO basis, standard NN potentials, which generate strong short range correlations, cannot be accommodated in the available basis space. So, we require renormalization methods that soften the interactions and make convergence faster with basis space enlargement. Okubo-Lee-Suzuki (OLS) renormalization method is one such method used in NCSM. In the present calculation, we have used two different realistic NN potentials, one is inside non-local outside Yukawa (INOY) that is taken from meson-exchange theory and the other is chiral next-to-next-to-next-to-leading order (N^{3}LO) potential which is a QCD based potential. In this approach, two variational parameters are used- HO frequency ($\hbar\Omega$) and $N_{\max}$.

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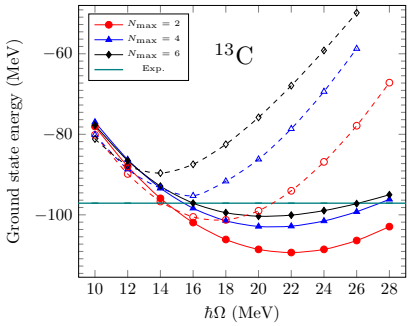

FIG. 1: Dependence of ground state energy of ^{13}C on HO frequency and basis space parameter $N_{\max}$ for INOY (solid lines) and N^{3}LO (dashed lines) interactions.

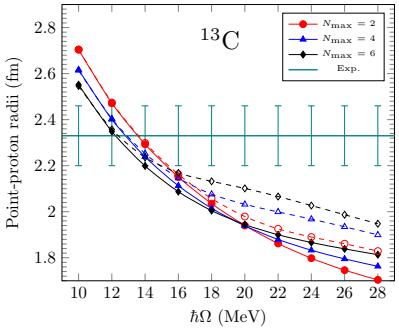

FIG. 2: Variation of point-proton radius of g.s., obtained with INOY (solid lines) and N^{3}LO (dashed lines) interactions in NCSM, for ^{13}C with the oscillator energy, $\hbar\Omega$ corresponding to different values of $N_{\max}$. Horizontal line represents the experimental data with error bar [5].

Results and discussion

Firstly, we have applied NCSM with INOY and N^{3}LO interactions to deduce the binding energy of ^{13}C. As we increase the basis space parameter, dimension of the Hamiltonian matrix increases exponentially due to involvement of more single-particle orbitals. For $N_{\max} = 2$ ($2\hbar\Omega$), the dimension comes out to be 21, which drastically rises up to 3.8×10^7 for $N_{\max} = 6$ in case of ^{13}C. So, NCSM calculations up to $6\hbar\Omega$ have been carried out in this work. In Fig. 1, we have plotted g.s. energy versus HO frequency for $N_{\max}$ in the range 2-6 $\hbar\Omega$. The frequency at which g.s. energy becomes minimum is known as optimal frequency, which is then used to calculate low-lying energy spectra and other spectroscopic properties. We have observed from Fig. 1 that dependence of $1/2^-$ state on HO frequency decreases with increment of the basis size. For largest $N_{\max}$ ($6\hbar\Omega$), the minimum g.s. energy is at $\hbar\Omega = 20$ MeV for INOY interaction. At this frequency, calculated NCSM g.s. energy is -100.344 MeV while experimental value is -97.108 MeV. For N^{3}LO interaction, the g.s. energy is -89.628 MeV at 14 MeV.

It has been found that observables like point-proton radius (r_p), transition strengths, etc. are also dependent on HO frequency. We have shown the variation of r_p with frequency for different values of $N_{\max}$ corresponding to INOY and N^{3}LO interactions for ^{13}C in Fig. 2. The converged radius is considered at a frequency where the radii curves for different $N_{\max}$ intersect. The estimated radius for N^{3}LO interaction is 2.20 fm which is close to the experimental value of 2.33(13) fm. For INOY interaction, converged radius is 1.96 fm. We can thus conclude that INOY interaction gives better results for energy spectra while N^{3}LO interaction is best suited for r_p. It should be noted that the optimal frequencies for g.s. energy and radius could be different.

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