Isotopic shift for magic nuclei

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1. Introduction

The isotopic shift is one of the standard procedure for studying the nuclear interaction. Although the older Skyrme interactions reproduce the nuclear matter (NM) as well as the finite nuclear properties, these forces fail to produce the experimental isotopic shift for Pb nuclei. After the success of the relativistic mean-field (RMF) theory of reproducing the bulk properties of finite nuclei, Sharma *et. al.* [1] tested the RMF results as input to analyze the isotopic shift of charge radius for Pb nuclei. However, these analysis are confined to N=126. Goddard *et. al.* [2] revisited the study and reproduced the isotopic shift at neutron number N = 126 for Pb-isotopes using advanced Skyrme Energy Density Functional. The recent experimental observation of Gorges *et. al.* [3] shows the isotopic shift for Sn at N=82. Thus, it is clear that the study of isotopic shift gives a better understanding of the structure of the nucleus.

2. Theoritical formalism

The calculated charge distribution radius is taken as the indicator for the determination of isotopic shift in the present analysis. Firstly, we highlight the ground state observable and compared with the experimental data wherever available to justify the parameters used. The well-known three-point method [3] is used to obtain the shift over the isotopic chain. The three-point formula can be expressed as,

$$\triangle_{kn}\mathcal{O}(Z,N) \equiv \frac{1}{2}[\mathcal{O}(Z,N+k) - 2\mathcal{O}(Z,N) + \mathcal{O}(Z,N-k)]. \quad (1)$$

Here $\mathcal{O}$ stands for the calculated value of the quantity, and $k = 2$ corresponds to curvature/kink parameter.

3. Results and discussion

In our present paper, the isotopic shift charge radius is calculated by relativistic mean-field (RMF) with G3 [4] & IOPB-I [5] sets for the magic nuclei Ca, Sn, Pb, and Z = 120 (predicted). We have performed the shift calculation using the three-point formula instead of concentrating the magic neutron to justify the results from Fig. 1. From the figure, one can find the peaks at the magic numbers N = 20, 28, and 40 for Ca isotopes, N=50, and 82 for Sn isotopes and N=126 for Pb. For Z = 120, the peak appears at N = 172, 184, along with two highly neutron-rich isotopes with neutron numbers N = 198 and 212 for the considered parameter sets showing the limitation of the spherical calculation. Hence, more systematic studies for the magicity of these two neutron-rich isotopes, namely, $^{318,332}120$, are highly welcome. Comparing the magnitude of peaks that appear for Z = 120, we find the large magnitude corresponding to the neutron numbers N = 172 and 184. Again it confirms the shell/sub-shell closure of $^{304}120$,

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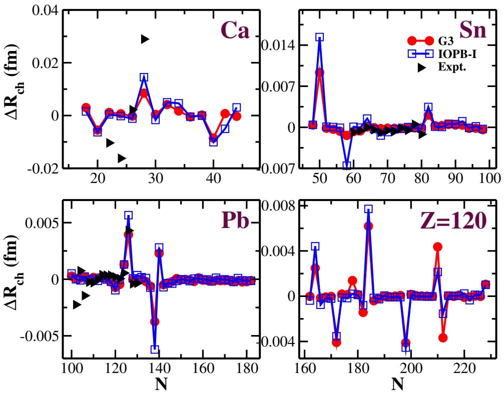

FIG. 1: The isotopic shift of charge radius ΔR_{ch} in fm for nuclei Ca, Sn, Pb, and Z = 120 are displayed as a function of neutron number using the three-point formula of Ref. [3] along with the experimental data [6].

which is predicted to be the next double magic nuclei in the superheavy valley [7] and reference therein.

4. Summary

In summary, although we are aware of the experimental isotopic shift of charge radius in Pb (N = 126) and Sn (N =82), a systematic theoretical analysis for such study are also presented for Ca and Z=120. The charge distribution is dominated by quantum shell effects for a shell/sub-shell closure nuclei/isotope [8], and that causes an irregularity in the systematic trend. In other words, the nuclei having large shell gap in single-particle energy levels, doubly magic nature, and with zero pairing energy always show the kinks and peaks in an isotopic chain [3]. Isotopic shifts are the portal to achieve the shell closure because they appear only when a large shell gap is maintained amongst the two adjacent orbitals, resulting in appearance of a sudden peak in the observable. Thus isotopic shift contributes a trivial aspect in finding the magicity/shell closure in the nuclear chart and vice-versa. A detailed version of our work can be found in [9].

Acknowledgments

One of the authors (JAP) is thankful to the Institute of Physics, Bhubaneswar, for providing computer facilities during the work. SERB partly reinforces this work, Department of Science and Technology, Govt. of India, Project No. CRG/2019/002691. MB acknowledges the support from FOSTECT Project No. FOSTECT.2019B.04, FAPESP Project No. 2017/05660-0, and the CNPq - Brasil.

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