Shape fluctuation model of ground state bands in Gd isotopes

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Introduction

In past years, many semi-classical models [1] for the energy levels of the ground-state bands in the even-even nuclei have been brought forward. The variable-moment of inertia (VMI) model [2] has been quiet successful in the fitting of the spectra of the deformed nuclei. Moreover, these classical models also predict the definite relation between the moment of inertia and the deformation. In this note we shall propose a shape fluctuation (SF) model that yields a better fit of the energy levels than the VMI model and also free from other unpleasant features that are present in the classical models.

In the microscopic description of the ground state rotational band, the energies of the various states are obtained by projecting out good-J states from an intrinsic wave function ϕ_k determined in the Hartee-Fock scheme [3]. The expression for the energy of the state having the angular momentum J is

$$E_J = \frac{\langle\phi_k|(P^J)^+ HP^J|\phi_k\rangle}{\langle\phi_k(P^J)^+(P^J)|\phi_k\rangle} \quad (1)$$

where P^J is the usual projection operator and H is the nuclear Hamiltonian. The subscript k has the value zero for the even-even nuclei and hence dropped from the equation. The Eq.(1) is shown to be equivalent to

$$E_J = E + BJ(J+1). \quad (2)$$

The constants E and B are functions of ϕ and are respectively the Hartee-Fock energy and inverse of twice the moment of inertia. The Eq.(2) can be written as

$$E_J = E(\phi(J)) + B(\phi(J))J(J+1). \quad (3)$$

making the Taylor's expression of $\phi(J)$ at $J = 0$, and relating only the first two terms in the expansion, one obtains from Eq.(3):

$$E_J = E\left(\phi_0 + J\phi_0'\right) + B\left(\phi_0 + J\phi_0'\right)J(J+1) \quad (4)$$

where $\phi_0 = \phi_{J=0}$ and $\phi_0' = \left.\frac{\partial\phi}{\partial J}\right|_{J=0}$. Again making the Taylor's expansion of E and B at $\phi_{J=0}$ one may write Eq.(4) as

$$\begin{aligned} E_J &= E(\phi_0) + J\phi_0' \left.\frac{\partial E}{\partial\phi}\right|_{\phi=\phi_0} + \cdots \\ &+ \left[B(\phi_0) + J\phi_0' \left.\frac{\partial E}{\partial\phi}\right|_{\phi=\phi_0} + \cdots\right] J(J+1) \\ &= E_0 + J\phi' E' + \cdots \\ &\quad + (B_0 + J\phi' B' + \cdots)J(J+1). \end{aligned} \quad (5)$$

keeping only the first order term in the expansion of E and B, in Eq.(5) one gets

$$\begin{aligned} E_J &= E_0 + J\phi' E' + (B_0 + J\phi' B')J(J+1) \\ &= B_0 J(J+1) + J\phi' E' + J\phi' B' J(J+1) \end{aligned} \quad (6)$$

In Eq.(6) the first order term gives the energy due to the rotation of the unfluctuated core whereas the second and the third term respectively give the excess in the intrinsic energy and rotation energy due to the fluctuation of the core.

Result and Discussion

For each nucleus, the second and third rows in Table I represent the level energies and the

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TABLE I: Experimental and calculated energies in keV of levels of ground-state bands of even-even nuclei.

$^{149}Gd(2)$			$^{149}Gd(5)$	
J	E_γ(Exp.)	E_γ(Cal.)	E_γ(Exp.)	E_γ(Cal.)
32.5	859	857	756	754
34.5	888	885	805	804
36.5	878	876	908	907
38.5	901	924	960	961
40.5	942	940	1016	1015
42.5	987	984	1070	1067
44.5	1033	1029	1126	1124
46.5	1081	1077	1183	1182
48.5	1130	1126	1241	1239
50.5	1181	1178	1299	1298
52.5	1233	1227	1358	1357
54.5	1286	1285	1417	1416
56.5	1340	1337	1476	1477
58.5	1395	1392	1535	1539
60.5	1451	1447	1595	1601
62.5	1507	1505		
64.5	1564	1562		
66.5	1620	1617		

fluctuation energies of the nuclei predicted in the SF model.
The parameters B_0, $\phi' B'$ and $\phi' E'$ obtained for each nucleus are given in Table II. Almost in all cases, the fit of the level energies with experimental in SF is a better agreement.

TABLE II: The value of parameters obtained from the least-squares fitting for the SD bands in Gd isotopes using the SF model.

Band	I_0	E_γ (Exp.)	E_γ (cal.)	B_0	$\phi' B'$	$\phi' E'$
$^{149}Gd(2)$	32.5	859	857	-6.392	0.299342	724.307
$^{149}Gd(4)$	34.5	726	724	23.762	-3.9e-04	-119.256
$^{149}Gd(5)$	32.5	756	754	18.091	0.105	33.102
$^{149}Gd(6)$	28.5	688	685	17.242	0.111	33.692

Conclusion

A systematic study of flat SD bands in Gd isotopes is made using the shape fluctuation model. The interband γ-transition energies of the SD bands in the Gd isotopes have been split into the rotational energy and shape fluctuation part.

References

[1] S.H. Harris, Phys. Rev. 138 (1965) B509: R. M. Diamond, F. S. Stephens and W. T. Swiatecki Phys. Letters 11 (1964) 315: J. E. Draper, D. G. McCauley and G. L. Smith, preprint: P.C. Sood, Contribution 2.12, Intern, Conf. on properties of nuclear states, Montreal 1969.

[2] M. A. J. Mariscotti, G.Scharff-Goldhaber and B. Buck, Phys. Rev. 178 (1969) 1864.

[3] A. Klein, R. M. Dreizler and T. K. Das, Phys. Letters 31B (1970) 333.