Possibility of Quantum Hall effect in the core of compact stars

Naman Pujari[1],[*] Akash Gupta[2], Jayanta Dey[1],
Aritra Bandyopadhyay[3,4], and Sabyasachi Ghosh[1]
[1]*Indian Institute of Technology Bhilai, GEC Campus, Sejbahar, Raipur 492015, Chhattisgarh, India*
[2]*Ludwig-Maximilians-Universitat, Theresienstrae 37, 80333 Munchen, Germany*
[3]*Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China and*
[4]*Guangdong-Hong Kong Joint Laboratory of Quantum Matter, Southern Nuclear Science Computing Center South China Normal University, Guangzhou 510006, China*

An extremely high densities and strong magnetic fields [1] can be expected in compact stars like white dwarf (WD) and neutron stars (NS). The values of electrical conductivity of the crustal matter act as an important input [2] for simulations of magnetized neutron stars in the general relativity, including binary magnetized neutron star mergers. It is due to the recently observed gravitational wave signal GW170817 [3], the binary neutron star merger simulation suddenly get special attention. Owing to this connection with these contemporary studies on neutron star merger and gravitational wave, the microscopic calculation of electrical conductivity in presence of magnetic field might be an important research topic, which is attempted in present work [4].

We know that in absence of magnetic field, the electrical conductivity along any direction remain same, which means that we will get an isotropic conductivity tensor $\sigma_{ij} = \delta_{ij}\sigma_D$. Here, Drude conductivity for relativistic quark matter can be expressed as $\sigma_D = \frac{nq^2\tau_c}{\mu}$, where $n = \frac{2g}{6\pi^2}(\mu^2 - m^2)^{3/2}$ is number density of quark with charge q, mass m, degeneracy factor $g = \text{spin} \times \text{color} \times \text{flavor} = 2 \times 3 \times 2 = 12$. The τ_c is relaxation time of quark, whose values are expected to be fermi (fm) order ($fm/c \sim 10^{-23}$ s).

In presence of external magnetic field (along z-direction), isotropic properties of conductivity tensor breaks down and we get different values of parallel (σ_{zz}) and perpendicular ($\sigma_{xx} = \sigma_{yy}$) components of electrical conductivity along with a new component - Hall conductivity ($\sigma_{xy} = -\sigma_{yx}$). Ohm's law for this picture will be

$$\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix} , \quad (1)$$

where

$$\sigma_{xx} = \sum_{f=u,d} \sigma_D \frac{1}{1 + (\tau_c/\tau_{Bf})^2} \quad (2)$$

$$\sigma_{yx} = -\sum_{f=u,d} \sigma_D \frac{\tau_c/\tau_{Bf}}{1 + (\tau_c/\tau_{Bf})^2} , \quad (3)$$

where $\tau_{Bf} = \mu/q_f B$ is cyclotron time scale and flavor degeneracy factor 2 will have to be removed from n or σ_D as they are summed now. By applying $\vec{E} = E_y\hat{y}$, repeatation of same calculation will give us the expressions of σ_{yy}, σ_{xy}, which follow the relations $\sigma_{xx} = \sigma_{yy}$, $\sigma_{xy} = -\sigma_{yx}$. Longitudinal conductivity along z-axis will remain unaffected by magnetic field and it will be $\sigma_{zz} = \sigma_D$.

Without magnetic field picture resistivity ρ_D will be exactly inverse of conductivity σ_D i.e. $\rho_D = 1/\sigma_D$ but in presence of magnetic field, we will get resistivity matrix, following

*Electronic address: npujari.hld@gmail.com

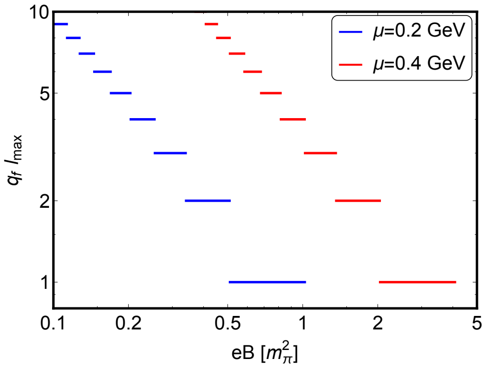

FIG. 1: Maximum Landau level $l^Q_{\rm max}$ as a function of magnetic field in quantized way for two values of chemical potential $\mu = 0.2$ GeV, $\mu = 0.4$ GeV.

inverse relation of Eq. (1):

$$\begin{pmatrix} \rho_{xx} & \rho_{xy} & \rho_{xz} \\ \rho_{yx} & \rho_{yy} & \rho_{yz} \\ \rho_{zx} & \rho_{zy} & \rho_{zz} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix} = \begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix} . \quad (4)$$

Comparing Eq. (1) and (4), we get the relations for each quark flavor:

$$\rho_{xx} = \rho_{yy} = \frac{\sigma_{xx}}{\sigma^2_{xx} + \sigma^2_{xy}} = \frac{1}{\sigma_D} = \frac{\mu}{n q_f^2 \tau_c},$$
$$\rho_{yx} = -\rho_{xy} = \frac{\sigma_{xy}}{\sigma^2_{xx} + \sigma^2_{xy}} = \frac{\tau_c}{\sigma_D \tau_B} = \frac{B}{n q_f} ,$$
$$\rho_{zz} = \frac{1}{\sigma_{zz}} = \frac{1}{\sigma_D} = \frac{\mu}{n q_f^2 \tau_c} . \quad (5)$$

According to these classical expressions of conductivity or resistivity tensors, one can draw their those components as functions of magnetic field B. Perpendicular component of conductivity σ_{xx} decreases with B, while its parallel component σ_{zz} remains independent of B as Lorentz force does not work along the direction of B. Hall conductivity σ_{xy} first increases and then decreases with B. In the case of resistivity tensor, parallel (ρ_{zz}) and perpendicular (ρ_{xx}) components are same and constant against B-axis, while Hall resistivity ρ_{xy} proportionally increases with B.

Now, if one introduces Landau quantization in the framework, then quantum expressions of resistivity and conductivity tensors can be obtained. In quantization picture, perpendicular momenta are quantized as $\vec{k}^2_\perp = 2lq_f B$, where l is Landau level, which can be maximum for zero Fermi-momentum along perpendicular direction. So, $l_{\rm max} = \frac{\mu^2 - m^2}{2q_f B}$, which is plotted in Fig. (1) for $\mu = 0.2$ and 0.4 GeV. Here, we can get a visualization of quantized Landau levels, whose maximum possible values will be increased as we decrease magnetic field. So, Landau level summation almost behave like integration in low B zone, where classical expressions can be restored from their quantum expressions but in high B zone, we can find quantized pattern of all conductivity/resistivity components including Hall, which popularly called quantum Hall effect (QHE). Present work has explored that possibility of quantum Hall effect (QHE), which will be presented in the conference with detailed mathematical descriptions.

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