Study of S-factors and thermonuclear reaction rate for p+^{13}N fusion reaction

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Introduction

The nuclear fusion at very low energies ($E \sim$ 1eV to few KeV) plays a crucial role in primordial nucleosynthesis of light elements. Nuclear fusion reaction in this energy regime can be explained successfully by quantum mechanical tunnelling through Coulomb barrier of interacting nuclei. In this work, we are focusing on the calculation of astrophysical S-factor and thermonuclear reaction rate by using selective resonant tunneling model (SRTM) for the p+^{13}N nuclear fusion reaction. Here we invoked complex Gaussian nuclear potential having a space varying imaginary part (see Eq. 1), to describe the absorption inside the nuclear well. The quantum mechanical findings for S-factors and reaction rate matches well with some experimental results.

Theoretical framework

Resonant tunneling in the light nuclear fusion reaction is sequentially followed by tunneling and decay. Selective resonance tunneling model (SRTM) is completely different from the well-known compound nucleus model (CNM) because in the former one, the penetrating particle may still remember its phase while in the latter one the penetrating particle looses memory of its history. SRTM selects both frequency of oscillation in the energy level and damping rate corresponding to the resonance absorption. In case of light nuclei the relative motion can be described by the wave function, $\psi(r,t)$ given by

$$\psi(r,t) = \frac{1}{\sqrt{4\pi}r}\phi(r)\exp(-i\frac{E}{\hbar}t)$$

where, $\psi(r,t)$ is the general solution of the Schrödinger wave equation for the interacting nuclei. The reaction cross-section in terms of the phase shift, δ_0 in the low energy limit (where only S-wave contributes) is given by $\sigma = \frac{\pi}{k^2}(1-|\eta|^2)$,where $\eta = e^{2i\delta_0}$ and k is the wave number corresponding to the relative motion. As we are considering here the complex nuclear potential

$$V_N(r) = -V_r \exp\left\{-\left(\frac{r}{\beta_r}\right)^2\right\} + iV_i \exp\left\{-\left(\frac{r}{\beta_i}\right)^2\right\} \quad (1)$$

the corresponding complex phase shift, δ_0 can be expressed as

$$cot(\delta_0) = W_r + iW_i \quad (2)$$

The fusion cross-section, σ can then be expressed as

$$\sigma = \frac{\pi}{k^2}\left\{-\frac{4W_i}{(1-W_i)^2+W_r^2}\right\} = \left(\frac{\pi}{k^2}\right)\left(\frac{1}{\chi^2}\right)\left\{-\frac{4\omega_i}{\omega_r^2+(\omega_i-\frac{1}{\chi^2})^2}\right\} \quad (3)$$

where, $\omega = \omega_r + i\omega_i = W/\chi^2 = (W_r + iW_i)/\chi^2$. The last factor in Eq. (3) within curly braces{} is called the astrophysical S-factor, which depends on the projectile energy, E and the cross-section, $\sigma(E)$. Real and imaginary parts of the nuclear potential, V_r and V_i determine the wave function inside the

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nuclear well where as the real and imaginary parts of complex phase shift, $(\delta_0)_r$ and $(\delta_0)_i$ give the Coulomb wave function outside the nuclear well. W_r and W_i, are introduced to make a linkage between the cross section and the nuclear potential, which facilitate the clear understanding of the resonance and selectivity in damping.

The Maxwellian-averaged thermonuclear reaction rate $< \sigma v >$ at some temperature T, is given by the following integral

$$< \sigma v > = \sqrt{\frac{8}{\pi\mu(K_B T)^3}} \int \sigma(E) E \exp\left(-\frac{E}{K_B T}\right) dE \quad (4)$$

where E is the center-of-mass energy, v is the velocity of relative motion and μ is the reduced mass of reactants.

Calculations and Results

Real and imaginary parts of depth and range are the controlling parameters in SRTM scheme. Resonating points are fine-tuned by adjusting these parameters. The astrophysical S-factor and thermonuclear reaction rate are computed by using Eq. (3) and Eq. (4) respectively. Plot of S-factor as a function of energy for p+^{13}N fusion reaction is shown in Fig.1, which matches pretty well with the experimental findings of Decrock *et. al.*, 1993[1]. Fig.2 depicts the temperature dependence of thermonuclear reaction rate, which indicates that a lateral shift occurs between computational and experimental results[1, 2] but agrees well after 600 KeV with NACRE-II [2]. The values of the adjustable parameters V_r, V_i, r_0, appearing in Eq. (1), for obtaining the final results are -26.99 MeV, -0.0872 MeV, 6.0996 fm respectively.

Summary and conclusion

As we consider SRTM here, according to which tunnelling probability itself depends upon the decay lifetime, thats why it is a single step process. The agreement with the experimental results for the deep sub-barrier fusion of light nuclei also suggests that the selective resonant tunnelling occurs in a single step. This indicates that the present method could be more fruitful for the study of light nuclear fusion reactions in the deep-sub barrier region.

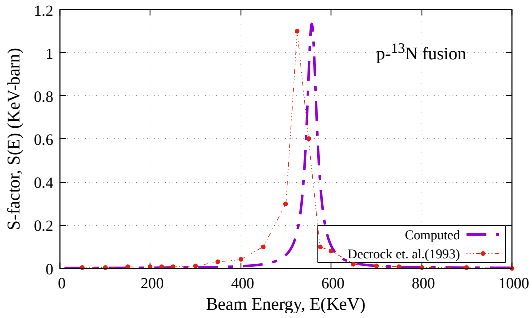

FIG. 1: Plot of S-factor (KeV-barn) as a function of energy (KeV) for p+^{13}N fusion reaction

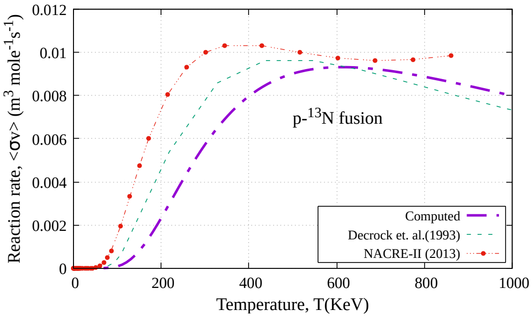

FIG. 2: Plot of reaction rate, $< \sigma v >$ $(m^3 mole^{-1} sec^{-1})$ as a function of temperature, T(KeV) for p+^{13}N fusion reaction

Acknowledgments

Authors acknowledge Prof. D. N. Basu of VECC Kolkata for a fruitful discussion and Aliah University for providing computational facilities.

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