A hadronic cascade is a superposition of several electron-photon sub-cascades initiated by neutral pions

Animesh Basak[1],* Tamal Sarkar[2], and Rajat K. Dey[1]
[1]*Department of Physics, University of North Bengal, Siliguri, West Bengal, INDIA 734013 and*
[2]*High Energy & Cosmic Ray Research Centre, University of North Bengal, Siliguri, West Bengal, INDIA 734013*

Introduction

Some recent works have revisited various conceptual issues on the shower age (s) of extensive air showers (EAS) initiated by hadrons [1]. Basically the shape of the lateral density distribution (LDD) of shower electrons (i.e. $e^{\pm}$) is indicated by the shower age in the EM cascade theory, and is fairly valid also for hadron/nuclei initiated showers. The s parameter is being estimated either from the reconstruction of EASs or from the radial variation of local shower age parameter (LAP)[1]. The work discusses how the different observed properties associated with s can be understood more precisely with simple analytical arguments and application of EAS simulations.

An analytic method

A hadron initiated shower is assumed to be a result of superposition of number of partial electron-photon sub-cascades started mostly from the decay of first generation π^os of the shower in the atmosphere. The LDD of electrons of a particular $e-\gamma$ sub-cascade (say, the i^{th} sub-cascade) is believed to be described by the NKG type function with an age parameter s_i. Usually the LDD of electrons of p/nuclei-initiated EASs can also be described by the NKG function but with a different shower age. Hence, the superposition principle applied to $e-\gamma$ sub-cascades in a hadron shower follows,

$$N_e C(s) X^{s-2}(1+X)^{s-4.5} = \sum_i |N_{e_i} C(s_i) X^{s_i-2}(1+X)^{s_i-4.5}| \quad (1)$$

where the symbols have their usual meanings. Let $\acute{s}$ be the lateral age of an equivalent EM cascade of the hadron shower initiated by primary e/γ. Dividing eq.(1) by $N_e C(\acute{s}) X^{\acute{s}-2}(1+X)^{\acute{s}-4.5}$, being the function describing the LDD of electrons of an equivalent EM cascade, one may then get the following,

$$s = \acute{s} - \frac{ln[C(s)/C(\acute{s})] - ln\ \sum_i \alpha_i C(s_i)/C(\acute{s}) h^{\delta_i}}{ln(h)} \quad (2)$$

with $\alpha_i = N_{e_i}/N_e$, $h = X(1+X)$ with $X = r/r_m$ and $\delta_i = s_i - \acute{s}$.

Using $C(s) \approx C(\acute{s}) \approx C(s_i)$, eq. (2) turns into,

$$s \approx \acute{s} + \frac{ln\ \sum_i \alpha_i h^{\delta_i}}{ln(h)} \quad (3)$$

Taking $N_{e_i} \approx n_e$ with $i = 1, 2, 3....n$), and $s_i \approx \tilde{s}$ for all sub-cascades. Let δ_i accounts the difference between the lateral shower ages of two EM cascades, in which one refers to the i^{th} $e-\gamma$ sub-cascade, and the rest is the effective EM cascade generated by primary e/γ. We have then,

$$s \approx \acute{s} + \frac{ln\ [(nn_e/N_e)h^{\delta}]}{ln(h)} \quad (4)$$

where $\delta = \tilde{s} - \acute{s}$ and $nn_e \approx N_e$. Then,

$$s \approx \acute{s} - \delta = 2\acute{s} - \tilde{s} \quad (5)$$

*Electronic address: animesh21@nbu.ac.in

In terms of LDDs of electrons for hadron- and e/γ-initiated showers, we can write,

$$(\acute{s} - \tilde{s}) \approx \frac{ln(n\alpha_e) - ln[\frac{\rho_{Had}(r)}{\rho_{EM}(r)}]}{ln[h]} \approx \delta \qquad (6)$$

δ can be estimated from the above relation by using simulations.

The LAP of a hadron- or e/γ-initiated showers is defined by

$$s^{Had}_{local}(i,j) = \frac{ln(F_{ij}X^2_{ij}Y^{4.5}_{ij})}{ln(X_{ij}Y_{ij})} \qquad (7)$$

For the equivalent EM cascade, the corresponding LAP is,

$$s^{EM}_{local}(i,j) = \frac{ln(\acute{F}_{ij}\acute{X}^2_{ij}\acute{Y}^{4.5}_{ij})}{ln(\acute{X}_{ij}\acute{Y}_{ij})} \qquad (8)$$

The superposition principle yields,

$$s^{Had}_{local}(i,j) \approx \frac{ln[(\sum_k \tilde{\rho}_{ij,k})\tilde{X}^2_{ij}\tilde{Y}^{4.5}_{ij}]}{ln(\tilde{X}_{ij}\tilde{Y}_{ij})} \qquad (9)$$

The minimum value from the LAP versus r curve is used as a lateral shower age of a shower. Consequently we have obtained; $s^{Had}_{local}(min) - \acute{s}^{EM}_{local}(min) \approx \delta$ and also the $\acute{s}^{EM}_{local}(min) - \tilde{s}_{local}(min) \approx \delta$.

Results and discussions

In the CORSIKA [2], the high-energy EPOS-LHC and low-energy UrQMD models are combined for generating p, e/γ and π^0 showers at $E = 2$ PeV. FIXCHI$\approx$ 75 gcm^{-2} is used for π^0 showers to deliver better results.

In Fig. 1 (top), we have plotted mean ρ_e versus r for various EASs. Agreement with the prediction by the analytic method is achieved. Analysis of data results, $\delta \approx s - \acute{s} = 0.053$ and $2\acute{s} - \tilde{s} = 0.899 \approx s$. When MC data are used in eq. (6), we have obtained, $\delta \approx \acute{s} - \tilde{s} \approx 0.05$.

The variation of LAP versus r is shown in Fig. 1 (bottom). The error of the LAP is found $\approx$ 0.04 for $12 < r < 205$ m. The minimum LAP from LAP versus r variation at about 50 m is taken as the lateral shower age of an EAS. We have obtained $\delta \approx s_{local}(min) - \acute{s}_{local}(min) \approx 0.044$ and $\delta \approx \acute{s}_{local}(min) - \tilde{s}_{local}(min) \approx 0.01$. We have noticed a rise in δ with r.

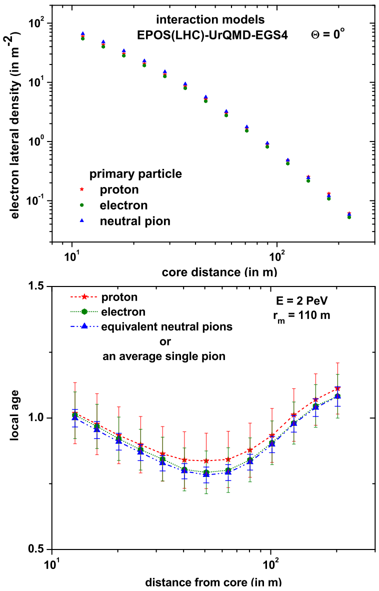

FIG. 1: Top: ρ_e versus r; Bottom: LAP versus r.

Conclusions

ρ_e versus r variations equivocally support the idea, explained in the adopted simple analytical argument. $\delta \approx s - \acute{s} = 0.053 \neq 0$ supports also the superposition principle. The value of δ is almost recovered in the language of LAP (i.e. $s_{local}(min) - \acute{s}_{local}(min)$) but the value $\acute{s}_{local}(min) - \tilde{s}_{local}(min)$ deviates much from its earlier value in terms of lateral shower age obtained from fitting procedure.

References

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