Thomas-Fermi Treatment for Hot and Magnetized White Dwarfs

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Introduction

The relativistic Feynman-Metropolis-Teller (FMT) treatment of compressed atoms is extended to treat magnetized matter. Each atomic configuration is confined by a Wigner-Seitz cell and is characterized by a positive electron Fermi energy. In the relativistic treatment, the limiting configuration is reached when the Wigner-Seitz cell radius equals the radius of the nucleus with a maximum value of the electron Fermi energy which cannot be attained in the presence of magnetic field due to the effect of Landau quantization of electrons within the Wigner-Seitz cell. This treatment is implemented to develop the equation of state for cold and magnetized white dwarf stars in the presence of Coulomb screening [1].

When a star is evolved through planetary nebulae to form a white dwarf (WD) it is very hot initially. Initially a WD radiates as a black body which has its peak near $\sim 10^8$ K. But it has no source of energy and therefore, the WD radiates and cools off gradually. The oldest WDs still radiate at temperatures $\sim 10^4$ K. In the present work, the properties of the hot WDs have been investigated in the presence of magnetic field. The effect of temperature has been incorporated following the procedure followed in Ref.-[2].

There are almost about 250 magnetized white dwarfs reported in the literature with well-determined fields and over $\sim$ 600 objects with uncertain fields. Surveys such as the Hamburg Quasar Survey, Sloan Digital Sky Survey and the Cape Survey have discovered these magnetized white dwarfs. The field distribution of magnetized white dwarfs is found to be in the range between 10^3 and 10^9 gauss, which basically provides the surface magnetic fields. However, due to the conservation of magnetic flux and high central density $\sim 10^{10}$ gm/cc, the central magnetic field can be several orders of magnitude higher than the surface magnetic field. In fact, the stability of WDs and the mass-radius relationship depends crucially on the central magnetic field. In the core of the highly magnetized WDs, the onset of electron captures and pycnonuclear reactions leads to local instabilities which thereby sets an upper limit ($\sim 10^{14}$ gauss) on the central magnetic field of the star [3].

Observations performed using ULTRACAM mounted on the New Technology Telescope on La Silla, Chile, William Herschel Telescope on La Palma, Spain, the Very Large Telescope at Paranal, Chile, ULTRA-SPEC mounted on the Thai National Telescope on Doi Inthanon, Thailand and X-shooter spectroscopy mounted at the Cassegrain focus of the VLT-UT2 at Paranal, Chile inferred that, for 16 WDs in detached eclipsing binaries with masses $\leqslant 0.48$ M$_\odot$, measured radii that are fully consistent with the He core models and 9% larger than the radii predicted in C-O core models. On the other hand, measured radii of WDs with masses $\geqslant 0.52$ M$_\odot$ are consistent with C-O core models [4].

Therefore, in the present work we take up the task to develop the equation of state treating WDs made up of different elements e.g. He, C, O, Fe subjected to finite temperature (as high as 10^8 K) and magnetic field (as high as 10^{14} gauss). We present some of the calculated results assuming uniform electron gas with no Coulomb correction due to interaction with the nuclear lattices and also present the formalism to incorporate the aforementioned correction.

Calculations and Results

Firstly, we summarize the formulation to treat the magnetized white dwarf matter at zero temperature considering the Coulomb correction to be small. Therefore, the Fermi distribution function $f(E) = 1$ for $E \leqslant E_F$ and $f(E) = 0$ otherwise. E_F is the Fermi energy or chemical potential μ of the system and given by,

$$E_F^2 = \mu^2 = p_F^2(\nu)\, c^2 + m_e^2 c^4 (1 + 2\nu B_D) \quad (1)$$

where, $p_F(\nu)$ is the Fermi momentum, with B_D being the magnetic field strength B in the unit of $B_c = \frac{m_e^2 c^3}{e\hbar} \approx 4.414 \times 10^{13}$ gauss. The electron mass is given by m_e, c is the speed of light and $\nu = n + \frac{1}{2} + \sigma$ with n being the Landau level quantum number and $\sigma = \pm\frac{1}{2}$, the spin quantum number. The number density of electrons is given by,

$$n_e = \frac{4\pi B_D}{h^3} m_e^2 c^2 \sum_{\nu=0}^{\nu_{max}} g_\nu p_F(\nu) \quad (2)$$

where, $g_\nu = 1$ for $\nu = 0$, $g_\nu = 2$ for $\nu > 0$ and ν_{max} is determined from Eq.-(1) imposing the condition that, $p_F \geqslant 0$. The energy density is given by,

$$\varepsilon_e = \frac{2\pi B_D m_e^2 c^2}{h^3} \sum_{\nu=0}^{\nu_{max}} g_\nu \left(E_F p_F(\nu) + x_F(\nu) \ln\left(\frac{E_F + p_F(\nu)}{\sqrt{x_F(\nu)}}\right)\right) \quad (3)$$

where, $x_F(\nu) = m_e^2 c^4 (1 + 2\nu B_D)$. The degeneracy pressure in the direction of the magnetic field is given by,

$$P_\parallel = \frac{2\pi B_D m_e^2 c^2}{h^3} \sum_{\nu=0}^{\nu_{max}} g_\nu \left(E_F p_F(\nu) - x_F(\nu) \ln\left(\frac{E_F + p_F(\nu)}{\sqrt{x_F(\nu)}}\right)\right) \quad (4)$$

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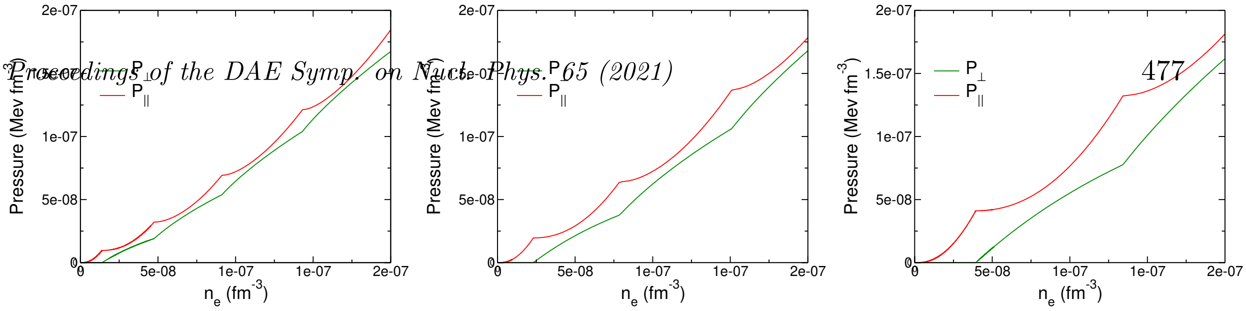

FIG. 1: Plots of Pressure vs. electron number density for B_D = 5, 7, 10 at a fixed temperature T = 10^5 K

The transverse component of the pressure is given by,

$$P_\perp = \frac{4\pi B_D^2 m_e^4 c^6}{h^3} \sum_{\nu=0}^{\nu_{max}} \nu g_\nu \ln\left(\frac{E_F + p_F(\nu)}{\sqrt{x_F(\nu)}}\right) \quad (5)$$

At finite temperature the Fermi distribution function is changed to $f(E) = 1/\left(1 + e^{\beta(E-\mu)}\right)$ and therefore, the electron density distribution yields to be,

$$n_e = \frac{2\pi}{h^3} m_e^2 c^2 B_D \sum_{\nu=0}^{\nu_{max}} g_\nu \int_{-\infty}^{+\infty} \frac{1}{\left(1 + e^{\beta(E-\mu)}\right)} dp_z \quad (6)$$

where, p_z is the $z-$component of momentum. The energy density ε_e, parallel pressure $P_\parallel$ and transverse pressure $P_\perp$ are given by,

$$\begin{aligned} \varepsilon_e &= \frac{2\pi B_D m_e^2 c^2}{h^3} \sum_{\nu=0}^{\infty} g_\nu \int_{-\infty}^{+\infty} \frac{E}{\left(1+e^{\beta(E-\mu)}\right)} dp_z \\ P_\parallel &= \frac{2\pi B_D m_e^2 c^2}{h^3} \sum_{\nu=0}^{\infty} g_\nu \int_{-\infty}^{+\infty} \frac{c^2 p_z^2}{E\left(1+e^{\beta(E-\mu)}\right)} dp_z \\ P_\perp &= \frac{2\pi B_D^2 m_e^4 c^6}{h^3} \sum_{\nu=0}^{\infty} \nu g_\nu \int_{-\infty}^{+\infty} \frac{1}{E\left(1+e^{\beta(E-\mu)}\right)} dp_z \end{aligned} \quad (7)$$

The equation of state for zero temperature with varying profile of magnetic fields can be computed using Eqs. (2-5). In Fig.-(1) numerically computed parallel and transverse components of pressure have been plotted against the electronic density n_e for a nonzero finite temperature of 10^5 K for finite number of Landau levels. The magnetization of the electron gas creates pressure anisotropy in the system. This anisotropy increases with the magnetic field and can be seen in Fig.-(1) that, the difference is increasing gradually from left to right. Therefore, it is expected that, the signature of the magnetic field in the mass-radius relationship of WDs will be vanishingly small for low magnetic field and large magnetic field will provide considerable changes in the mass-radius relationship i.e. in presence of the magnetic field, the shape of the WD will change from spherical to prolate or oblate ellipsoid depending on the direction of the magnetic field. The mass-radius relationship will be studied in the context of both Newtonian and Einsteinian theory of gravitation to quantify the qualitative inferences made here.

The results shown in the Fig.-(1) have been calculated ignoring the Coulomb correction due to the interaction between nucleus and the electron cloud. To incorporate the Coulomb correction, we follow the procedure developed in Ref.-[1]. In this case, the WD will be considered to be made up of atoms and each individual atom is confined within a cell, called Wigner-Seitz cell, which has a positively charged nucleus at its center surrounded by a relativistic gas of electrons whose number density varies with radial distance having maximum near the center and minimum on the surface. The Wigner-Seitz cell at zero temperature is characterized by Fermi energy E_F given by,

$$E_F = \sqrt{p_F^2(\nu)c^2 + m_e^2 c^4 (1 + 2\nu B_D)} - eV(r) \quad (8)$$

where, $V(r)$ is the Coulomb potential an electron is subjected due to the interaction between the nucleus and the electron gas. In presence of the potential $V(r)$, the number density of electron gas is modified according to the Poisson equation,

$$\nabla^2 V(r) = -4\pi e\left(n_p(r) - n_e(r)\right) \quad (9)$$

where, $n_p(r)$ is the constant density of proton confined within the nuclear radius and the electron density $n_e(r)$ is given by Eq.-(2) when temperature is zero. The electron density $n_e(r)$ is governed by Eq.-(6), when finite temperature is considered. Eq.-(9) can only be solved numerically and requires significant computation while magnetic field is involved. Once the solution for electron density distribution $n_e(r)$ is found for different densities, field strengths and temperatures, the equation of state for hot and magnetized WD matter can be developed.

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