Perturbative and Non-perturbative Instanton Induced contributions to D-meson masses

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Introduction

Rapid developments in the high-energy experimental facilities provide numerous states in the heavy-light sector. In the last decade observations of various excited states by $LHCb$ and $BABAR$ collaboration has enriched the spectrum of $D(c\bar{q}, q = u, d)$ mesons. On the theoretical side, phenomenological potential model approach provides suitable and effective way to understand underlying dynamics of mesons. In the present work, we examine perturbative and non-perturbative (short range and long range) instanton effects in the shift of the spin average mass of S wave D mesons.

Theoretical Formalism

The properties of hadrons can be systematically described by the non-relativistic potential models. The popular Cornell potential can be represented as

$$V_{Cornell}(r) = \frac{-4\alpha_s}{3r} + \sigma r \quad (1)$$

where first term represents short-distance Coulomb like attractive part dominated by the central part of the OGE and second term represents linear scalar confining part at large distances. Here, σ is the string constant, a pure phenomenological parameter. QCD Vacuum exhibits rich topological structures and one of them is instantons. In simple terms, instantons are known as the large fluctuation of the gluon field and tunneling events from one minima of the energy to other. To understand the effects of instantons on hadron spectrum we consider Instanton Induced Interaction (III) potential within the Instanton liquid model (ILM) framework, in addition to Cornell potential. The average size of instanton $(\bar{\rho})$ and the average distance between instantons $(\bar{R})$ are two important parameters of ILM. Different approach suggests the different set of parameters of these values. For the present study we have used 0.36 fm for $(\bar{\rho})$ and 0.89 fm for $(\bar{R})$. The instanton potential is also defined for two regions of quark anti-quark interactions. These regions are separated by instanton size. One region is when $r \gg \bar{\rho}$; distance between quark and anti-quark is larger than the size of the instanton and non-perterbative aspect of the quark anti-quark interaction. The instanton induced interaction in this region can be written as [1–3]

$$V_{III}^{\mathcal{NP}}(r) \simeq \frac{4\pi\bar{\rho}^3}{\bar{R}^4 N_c}\Big[\mathcal{I}_0^{\mathcal{N}}\Big\{1 + \sum_{i=1}^{2}[a_i^{\mathcal{N}} x^{2(i-1)} + a_3^{\mathcal{N}}(-b_3^{\mathcal{N}} x)^i)]e^{-b_i^{\mathcal{N}} x^2} + \frac{a_3^{\mathcal{N}}}{x}(1 - e^{-b_3^{\mathcal{N}} x^2})\Big\}\Big] \quad (2)$$

where the quantity inside the bracket is parametrization of the function $\mathcal{I}^{\mathcal{N}}(x)$ with the parameters $\mathcal{I}_0^{\mathcal{N}} = 4.4162$, $a_1^{\mathcal{N}} = -1$, $a_2^{\mathcal{N}} = 0.1287$, $a_3^{\mathcal{N}} = -1.1047$, $b_1^{\mathcal{N}} = 0.4048$, $b_2^{\mathcal{N}} = 0.4539$, $b_3^{\mathcal{N}} = 0.4207$ [2]. The other region is when $r \ll \bar{\rho}$; distance between the quark interquark is smaller than the size of instantons. This region is known as the perturbative part of the quark anti-quark interaction. And the instanton induced interaction here according to ILM can be given as [1–3]

$$V_{III}^{\mathcal{P}} = \frac{-4\alpha_s}{3}(f_{scr} - 1) \quad (3)$$

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TABLE I: S wave spin-average masses of D ($c\bar{q}$, $q = u, d$) mesons (in MeV). $m_c = 1.27$ GeV $m_{u,d} = 0.003$ GeV and $\sigma = 0.14\ GeV^2$.

n	$V_{Cornell}$	$V_{III}^{\mathcal{P}}$	$V_{III}^{\mathcal{NP}}$	V_{III}^{Total}
α_s =0.2				
1	1974	41	57	98
2	2631	32	89	121
3	3138	25	122	147
4	3624	17	161	178
5	4088	11	186	197
α_s =0.4				
1	1852	83	48	131
2	2549	66	81	147
3	3084	51	116	167
4	3579	36	154	190
5	4059	21	184	205

where the screening function f_{scr} is expressed as

$$f_{scr}(x) = 1 - \frac{2x}{\pi}\Big[\mathcal{I}_0^{\mathcal{P}}\Big\{\sum_{i=1}^{2}[a_i^{\mathcal{P}} x^{2(i-1)} + a_3^{\mathcal{P}}(-b_3^{\mathcal{P}}x)^i)]e^{-b_i^s x^2} + \frac{a_3^{\mathcal{P}}}{x}(1 - e^{-b_3^{\mathcal{P}}x^2})\Big\}\Big] \quad (4)$$

where, $\mathcal{I}_0^{\mathcal{P}} = 0.7439$, $a_1^{\mathcal{P}} = 1$, $a_2^{\mathcal{P}} = -0.0362$, $a_3^{\mathcal{P}} = 2.1181$, $b_1^{\mathcal{P}} = b_3^{\mathcal{P}} = 0.3293$, $b_2^{\mathcal{P}} = 0.4605$[2].

The numerical solutions of the bound spectra of D mesons is obtained by solving the Schrödinger equation with incorporation of Cornell and Instanton induced potentials for different regions. The trial wave function used for present study is given as

$$R_{nl}(r) = \left(\frac{n!}{\alpha^{2l(2l+3)}(n + 2l + 2)}\right)^{\frac{1}{2}} (\alpha r)^l \exp(-\alpha r/2)\, \mathrm{L}_n^{2l+2}(\alpha r) \quad (5)$$

Results and Discussion

In the present study we predict the spin average mass of the S wave D mesons using the Cornell potential. Our computed results are presented in Table 1. We fit the ground state spin average mass of 1974 MeV from the PDG listed ground state of D mesons ($D^*(2010)$ and $D(1867)$) [4]. Our aim is to know how much shift induce in the spin average mass due to inclusion of instanton effects. The qualitative comparison of the results will be done by the inclusion of the spin dependent interactions but here we focus on spin-average masses of ground and few radial excited states only. We also represent the effect of the instanton induced interaction in perturbative ($r \ll \bar{\rho}$) and non-perturbative ($r \gg \bar{\rho}$) region on the spin-average mass of D mesons. As the potential $V_{III}^{\mathcal{NP}}$ and $V_{III}^{\mathcal{P}}$ are positively defined, their contribution is positive to the mass spectrum. The QCD instanton effects and the addition of the strong confinement are important to describe the phenomena in the non-perturbative region in QCD [3]. But the effect of instantons in perturbative region is not known well. Within QCD based models, the underlying dynamics in the perterbative region canbe explored by varying the strong coupling constant α_s. So, we present spin average mass for two values of α_s as 0.2 and 0.4. The contribution of the instantons to the Cornell potential is found to be 4.79 % for $\alpha_s = 0.2$ and 5.72 % for $\alpha_s = 0.4$. We can see that the contribution of $V_{III}^{\mathcal{P}}$ increases two times if the value of α_s increases. This is due to the fact that the $V_{III}^{\mathcal{P}}$ is directly proportional to α_s. While the $V_{III}^{\mathcal{NP}}$ have small effects on mass due to increasing α_s as it is independent of α_s. The small change in $V_{III}^{\mathcal{NP}}$ corresponds small variation in wave function. Also, One can see that the $V_{III}^{\mathcal{NP}}$ contribution increases towards excited states while on contrary $V_{III}^{\mathcal{P}}$ contribution decreases. The total instanton contribution increases towards the excited states. Also, on increasing α_s the total instanton contribution affects ground state significantly while in higher excited state its contribution is just few MeV only. In summary, we emphasize that the instanton contribution to the whole spectrum is just few percentage but important in the finetuning of the mass spectra.

References

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