M1 and E1 transitions of heavy-light quarkonia

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Introduction

The radiative transitions of heavy quarkonia are of considerable experimental [1], and theoretical interest, and provide an insight into the dynamics of quarkonium. Electric dipole E1 transitions are much stronger than the magnetic dipole, M1 transitions, though the M1 are more sensitive to relativistic effects. For details of M1 and E1 transitions, see [2-3].

In this work, we focus on the radiative decays of the charmed and bottom vector mesons through the processes, $V \to P\gamma$, $V \to S\gamma$, and $S \to V\gamma$, and $A^- \to P\gamma$, where V, P, S and A^- refer to vector, pseudoscalar, scalar and axial quarkonia, 1^{+-}. These transitions involve quark-triangle diagrams with two hadron vertices. We have recently expressed the transition amplitude M_{fi} as a linear superposition of terms involving all possible combinations of $++$, and $--$ components of Salpeter wave functions of final and initial hadron, with coefficients being related to results of pole integrations over complex σ-plane [2]. We have calculated the above M1 and E1 transitions. We have used algebraic forms of Salpeter wave functions obtained through analytic solutions of mass spectral equations for ground and excited states of $0^{++}, 1^{--}$, and 0^{-+} heavy-light quarkonia [3–5] in approximate harmonic oscillator basis to calculate their decay widths. The input parameters used by us were obtained by fitting to their mass spectra. We have compared our results with experimental data and other models, and found reasonable agreements.

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Radiative decays of heavy-light quarkonia through $V \to P\gamma$

To apply the framework of BSE to study radiative decays, $V \to P\gamma$, we have to remember that there are two Lorents frames: the rest frame of the initial meson, and the rest frame of final meson. We first write relationship between the momentum variables of the initial and final meson. Here, P, and q are the total momentum and the internal momentum of initial hadron, while P', and q' are the corresponding variables of the final hadron, and let k, and $\epsilon^{\lambda'}$ be momentum and polarization vectors of emitted photon, while ϵ^{λ} be the polarization vector of initial meson. Thus if $p_{1,2}$, and $p'_{1,2}$ are the momenta of the two quarks in initial and final hadron respectively.

We decompose the internal momentum q of the initial hadron into two components, $q = (\hat{q}, iM\sigma)$, where $\hat{q}_\mu$ is the component of internal momentum transverse to P such that $\hat{q}.P = 0$, while σ is the longitudinal component in the direction of P. Since we study the process in the frame of initial hadron, we decompose the internal momentum, q' of final meson into two components $q' = (\hat{q}', iM\sigma')$, with $\hat{q}' = q' - \sigma' P$ transverse to initial hadron momentum, P, and $\sigma' = \frac{q'.P}{P^2}$, longitudinal to P. Thus, $P.\hat{q}' = 0$. We now first try to find the relationship between the transverse components of internal momenta of the two hadrons, $\hat{q}$, and $\hat{q}'$. In the rest frame of initial meson, we can relate the components of internal momenta of the two hadrons as, $\hat{q}' = \hat{q} + \hat{m}_2 \hat{P}'$, and $\sigma' = \sigma + \alpha$, where, $\alpha = \hat{m}_2 \frac{M'^2 - M^2}{2M^2}$. The EM transition amplitude of the process is

$$M_{fi} = -i \int \frac{d^4q}{(2\pi)^4} Tr[e_q \overline{\Psi}_P(P', q') \not{\epsilon}' \Psi_V(P, q) \\ S_F^{-1}(-p_2) + e_{\overline{Q}} \overline{\Psi}_P(P', q') S_F^{-1}(p_1) \Psi_V(P, q) \not{\epsilon}'],$$

where M_{fi} is written in the rest frame of the initial hadron. Here, the first term corresponds to the first diagram, where the photon is emitted from the quark (q), while the second term corresponds to the second diagram where the photon is emitted from the antiquark ($\overline{Q}$) in vector meson. The transition amplitude, M_{fi} for the process, $V \rightarrow P\gamma$ in terms of the transition form factor, F_{VP} is expressed as [2],

$$M_{fi} = F_{VP}\ \epsilon_{\mu\nu\alpha\beta} P_{\mu} \epsilon_{\nu}^{\lambda'} \epsilon_{\alpha}^{\lambda} P'_{\beta},$$

where the antisymmetric tensor, $\epsilon_{\mu\nu\alpha\beta}$ ensures its gauge invariance. The decay width, Γ in turn can be expressed as [2], $\Gamma = \frac{\alpha_{e.m.}}{3}|F_{VP}|^2\omega_k^3$, where, ω_k is the kinematically allowed energy of the emitted photon. Decay widths for $V \rightarrow P\gamma$ calculated in [2] along with experimental data are listed in Table 1

Radiative decays of heavy-light quarkonia through $V \rightarrow S\gamma$

The amplitude, M_{fi} for this process can be finally expressed as,

$$M_{fi} = S_1(\epsilon^{\lambda'}.\epsilon^{\lambda}) + S_2\beta(\epsilon^{\lambda'}.P)(\epsilon^{\lambda}.P'),$$

$$S_1 = -ieN_S N_V \frac{1}{M^2}\int \frac{d^3\hat{q}}{(2\pi)^3}\frac{\phi_S(\hat{q}')\phi_V(\hat{q})}{16\omega_1\omega_2\omega'_1\omega'_2}\Theta_1,$$

$$S_2 = -ieN_S N_V \frac{1}{M^2}\int \frac{d^3\hat{q}}{(2\pi)^3}\frac{\phi_S(\hat{q}')\phi_V(\hat{q})}{16\omega_1\omega_2\omega'_1\omega'_2}\Theta_2,$$

where, S_1, and S_2 are the form factors, with detailed expressions in [2]. We give the results of M1 transition, $V \rightarrow P\gamma$, and E1 transition, $V \rightarrow S\gamma$ obtained in BSE framework in Table 1 along with experimental data[1].

Radiative $A^- \rightarrow P\gamma$ decays

After a series of steps, we can express the transition amplitude, M_{fi} in terms of a single form factor, S_1, whose expression is,

$$M_{fi} = S_1\left[(\epsilon'.\epsilon) - \frac{1}{\bar{P}.k}(\bar{P}.\epsilon')(k.\epsilon)\right];$$

$$S_1 = -ieN_A N_P \frac{1}{M^2}\int \frac{d^3\hat{q}}{(2\pi)^3}\frac{\phi_P(\hat{q}')\phi_A(\hat{q})}{16\omega_1\omega_2\omega'_1\omega'_2}$$

$$(\Theta_1(\hat{q}^2) + \Theta'_1(\hat{q}^2))$$

where, $\bar{P} = P + P'$ is the sum of momenta of initial and final mesons and $k = P - P'$ is the emitted photon momentum. The expressions for form factors, $\Theta_1(\hat{q}^2)$, and $\Theta'_1(\hat{q}^2)$ in the expression for M_{fi} above are given in detail in [3]. We then calculate the decay width, Γ. The aim of doing this study was to mainly test our analytic forms of wave functions obtained as solutions of mass spectral equations in an approximate harmonic oscillator basis obtained from 4 × 4 BSE as a starting point, that has so far given good predictions [2-5] not only of the mass spectrum of heavy-light quarkonia [4-6], but also their various transitions [2-6], such as leptonic decays, two photon decays, and single photon radiative decays.

TABLE I: Radiative decay widths of heavy-light mesons (in Kev) for M1 and E1 transitions calculated in BSE, along with experimental data

	BSE-CIA[2, 3]	Expt.[1]
$\Gamma_{J/\psi(1S_1)\rightarrow\eta_c(1S_0)\gamma}$	1.7035	1.5793±0.0112
$\Gamma_{\psi(2S_1)\rightarrow\eta_c(2S_0)\gamma}$	0.1820	0.2002±0.008
$\Gamma_{D^*(1S_1)\rightarrow D(1S_0)\gamma}$	1.2843	1.3344±0.0072
$\Gamma_{D^*(2S_1)\rightarrow D(2S_0)\gamma}$	0.1381	
$\Gamma_{B_c^*(1S_1)\rightarrow B_c(1S_0)\gamma}$	0.0664	
$\Gamma_{\psi(2S_1)\rightarrow\chi_{c0}(1P_0)\gamma}$	33.3985	28.5714±0.0432
$\Gamma_{\psi(3S_1)\rightarrow\chi_{c0}(2P_0)\gamma}$	61.6924	
$\Gamma_{\psi(3S_1)\rightarrow\chi_{c0}(1P_0)\gamma}$	1.815	
$\Gamma_{D^*(2S_1)\rightarrow D(1P_0)\gamma}$	1.0214	
$\Gamma_{B_c^*(2S_1)\rightarrow B_c(1P_0)\gamma}$	9.6489	
$\Gamma_{h_c(1P)\rightarrow\eta_c(1S_0)\gamma}$	363.047	357±204
$\Gamma_{h_c(2P)\rightarrow\eta_c(2S_0)\gamma}$	187.145	
$\Gamma_{h_c(2P)\rightarrow\eta_c(1S_0)\gamma}$	20.195	
$\Gamma_{\eta_c(2S_0)\rightarrow h_c(1P)\gamma}$	6.909	

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