QCD sum rule analysis of Bottomonium ground states

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Introduction

Bottomonia are the bound states of a heavy bottom quark (b) and its anti-quark ($\bar{b}$). In-medium masses of the Bottomonium ground states [$1S(\Upsilon(1S), \eta_b)$ and $1P(\chi_{b0}, \chi_{b1})$], are investigated in strongly magnetized nuclear medium within the QCD sum rule (QCDSR) framework. In-medium masses are obtained through the medium modified scalar and twist-2 gluon condensates which are calculated in the chiral SU(3) model. Huge magnetic fields have been estimated in the non-central, ultra relativistic, heavy ion collision experiments at RHIC (BNL), LHC (CERN) [1]. Study of the effects of density, isospin asymmetry, magnetic fields on the hadronic properties have various important observable consequences in such collisions.

Masses within QCDSR

QCD sum rules are based on the current correlation function ($\Pi_{\mu\nu}(q)$) and the dispersion relation. On its phenomenological side, Im$\Pi \sim$ hadronic spectral density, which consists of a pole and a continuum. The theoretical side is derived in the deep Euclidean region ($Q^2 \equiv -q^2 >> 0$), $\Pi(q^2)$ can be expanded via operator product expansion (OPE), as $\Pi(q^2) = \sum_n C_n < O_n >$; $< O_n >$ are associated with the QCD condensates and C_n are the Wilson coefficients [2]. The non perturbative nature of QCD, thus enters into QCDSR through these condensates. By dispersion relation the OPE side is connected to the phenomenological side. QCDSR, thus provides a direct link between the various hadronic observables (masses, decay widths etc.) through spectral density to the fundamental QCD quantities. Thus, the in-medium hadronic properties can be obtained through the medium modifications of the condensates. The continuum part in the spectral density is to be neglected by the moment sum rule. In the moment sum rule, additional derivatives are taken through moments, and the ratio of the (n-1)th moment (M_{n-1}), to the nth moment (M_n) are being equated on both sides. Inserting Im$\Pi(s) = f_0\delta(s - m^2) +$ corrections [3], the in-medium mass squared of the i-type bottomonium ground state (i = vector, pseudoscalar, scalar and axial vector) is given by [2]

$$m_i^{*2} \simeq \frac{M_{n-1}^i(\xi)}{M_n^i(\xi)} - 4m_b^2\xi \qquad (1)$$

where ξ = renormalization scale, $m_b(\xi)$, running bottom quark mass [2]. In the OPE side, $M_n(\xi)$ can be written as [2],

$$M_n^i(\xi) = A_n^i(\xi)[1 + a_n^i(\xi)\alpha_s + b_n^i(\xi)\phi_b + c_n^i(\xi)\phi_c] \qquad (2)$$

Here, $\alpha_s(\xi)$ is the running coupling constant and A_n^i, a_n^i, b_n^i, c_n^i are the Wilson coefficients [2]. The in-medium masses of the bottomonium ground states are thus coming from the medium modified scalar and twist-2 gluon condensates through ϕ_b and ϕ_c respectively and the medium independent Wilson coefficients for the respective channel [2, 4].
The spin-mixing effects on the $\Upsilon^{||}(1S)$ and η_b states in the presence of magnetic fields, are given by [5]

$$m^{eff}_{\Upsilon(1S),\eta_b} = m^*_{\Upsilon(1S),\eta_b} \pm \Delta m_{sB} \qquad (3)$$

Here, $m^*_{\Upsilon(1S),\eta_b}$ are the in-medium masses of the 1S bottomonia calculated within QCDSR,

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and Δm_{sB} is the shift due to spin-magnetic field interaction,

$$\Delta m_{sB} = \frac{\Delta M}{2}\left((1+{\chi_{sB}}^2)^{1/2}-1\right) \quad (4)$$

with $\chi_{sB} = \frac{2g\mu_b B}{\Delta M}$; $\mu_b = (\frac{1}{3}e)/(2m_b)$, bottom quark Bohr magneton, $m_b = 4.7$ GeV, constituent bottom quark mass; $\Delta M = (m^*_{\Upsilon(1S)} - m^*_{\eta_b})$, g =2 (ignoring the effects of the anomalous magnetic moments of the b and $\bar{b}$).

Gluon condensates are associated with the trace anomaly of QCD within the chiral $SU(3)$ model [6]; ϕ_b and ϕ_c are calculated in terms of the scalar dilaton field, χ, which mimics the broken scale invariance of QCD through a logarithmic potential, and the non-strange, σ, strange, ζ, isovector, δ scalar fields through the explicit chiral symmetry breaking term [4, 7].

Results and Discussions

The in-medium masses of the bottomonium ground states are observed to decrease with increasing density of the medium, P-wave states showing larger modifications than the S-wave states. The effects of isospin asymmetry as well as magnetic fields are pronounced at higher density although these effects are not so prominent. The magnetic fields contribution are taken through the Landau levels of protons and the anomalous magnetic moments of both protons and neutrons of the nuclear medium in chiral SU(3) model Lagrangian. The magnetic fields have important contributions on the in-medium masses through the spin-magnetic field interaction between the longitudinal component of the spin-triplet, $\Upsilon^{||}(1S)$ and spin-singlet state, η_b of lowest lying S-wave bottomonia. The study shows an interesting rise (drop) of the in-medium masses of $\Upsilon^{||}(1S)$ (η_b) with increasing magnetic fields, which might be observed as a quasi-peak at m_{η_b} in the dilepton spectra at RHIC and LHC where huge magnetic fields have been generated in peripheral heavy ion collisions.

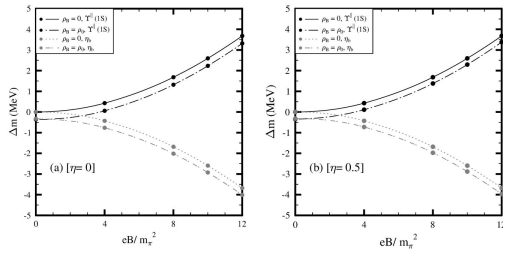

FIG. 1: Mass shifts (MeV) are plotted as a function of magnetic field, eB (in units of m_π^2) for $\Upsilon^{||}(1S)$ and η_b by considering the effects of spin-mixing in the presence of magnetic fields

TABLE I: Masses (MeV) are shown at $\rho_B = \rho_0, 2\rho_0$, and asymmetry parameter, $\eta = 0.5$ for magnetic fields of $4m_\pi^2$ and $12m_\pi^2$

mass	ρ_0		$2\rho_0$	
	$4m_\pi^2$	$12m_\pi^2$	$4m_\pi^2$	$12m_\pi^2$
	$\eta = 0.5$	$\eta = 0.5$	$\eta = 0.5$	$\eta = 0.5$
m_{η_b}	9681.18	9681.19	9680.85	9680.91
$m_{\Upsilon(1S)}$	9750.93	9750.95	9750.45	9750.55
$m_{\chi_{b0}}$	10572.65	10572.69	10571.40	10571.66
$m_{\chi_{b1}}$	10811.36	10811.40	10810.13	10810.39

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