Analysis of Ground State Wave Function of Effective Hulthen Potential*

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Introduction

The Hulthen potential is one of the important short-range potentials in physics. General wave functions of this potential have been used in solid-state and atomic physics In the recent times, there have been increasing interests in finding the analytical solutions of wave equations in relativistic and non-relativistic quantum mechanics such as Schrdinger spin-less Bethe Salpeter equations with different potential models problems.

Theoretical Background

The Hulthen potential V_H is defined as the

$$V_H(r) = -Ze^2\mu \frac{\exp(\frac{-r}{\mu})}{1-\exp(\frac{-r}{\mu})} \tag{1}$$

The radial part of Schrdinger wave equation for the relative motion of two particles interacting via Hulthen potential can be written as

$$\Big[-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{d}{dr}\Big(r^2\frac{d}{dr}\Big)+V_{eff}(r)\Big]u(r) = Eu(r) \tag{2}$$

Making the standard change $R(r) = r^{-1}u(r)$.

$$\Big[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+V_{eff}(r)\Big]u(r) = Eu(r) \tag{3}$$

The $V_{eff}(r)$ is the effective potential, which consists of generalized Hulthen potential $V_H(r)$ and the centrifugal term. The equation 3 can not be solved analytically due to the centrifugal term, we have to use a proper approximation of this term.The equation 3 allow us to obtain,

$$\Big[-\frac{\hbar^2}{2\mu}\frac{d}{dx^2}+V_{eff}(x)\Big]u(x) = Eu(x) \tag{4}$$

Where

$$V_{eff}(r) = -Ze^2\delta\frac{e^{-x}}{1-e^{-x}} + \frac{l(l+1)\hbar^2\delta^2}{2\mu}\Big[c_0+\frac{c_1e^{-x}}{(1-e^{-x})}+\frac{c_2e^{-2x}}{(1-e^{-x})^2} \tag{5}$$

Where $\delta r = x$, satisfying the boundary condition at z=0 and z=1, In terms of Greens function g(z, z). the solution of Equation 5 will be obtained by solving the integral equation and is integrable under the exchange of variables $z = e^{-x}$. By definition of Greens function

$$\Big[\frac{d^2}{dz^2}+\frac{1}{z}\frac{d}{dz}-\frac{l(l+1)c_2z^2+c_2}{(1-z)^2}+\lambda^2\Big]g(z,z') = -\delta(z-z')\frac{1}{z'(1-z')} \tag{6}$$

Under the boundary condition $\lim_{z\to 0} z(z-1)g(z-z') = 0$, equation 6 easily handled using the following supposition

$$u(z,z') = z^{\lambda}(1-z)^{\beta}g(z,z') \tag{7}$$

The radial wave function can be found by integrating equation

$$u(z) = \alpha\int_0^1 g(z,z')u(z')\frac{1}{z'(1-z')}dz' \tag{8}$$

iteratively. As $u(z)$ also appear on the right-hand side of equation 8, therefore, we propose its value under the restriction that it must be finite everywhere from $z = 0$ and $z = 1$. According to equation 8 the radial wave-function has the form $z'^{\lambda+1}$ nearly z equals to zero and has the form of $(1-z)^{\beta+1}$ near z equals to one. Therefore in the present case an appropriate choice for the appropriate value of $u(z')$ will be,

$$u(z') = z'^{\lambda+1}(1-z')^{\beta+1} \tag{9}$$

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With equation 9 the wave-function remains finite in the range zero to one. However, the exact solution can be found by solving equation 8 iteratively. The proposed wave-function equation and Green's function $g(z,z')$ are inserted into equation 8 and the integration is carried out to obtain the solution $u(z)$. In the next iteration the resultant wave $u(z)$ is considered as a proposed wave-function to get another solution. The process is repeated several times for a better approximation to the standard solution given by equation 7.

Results and Discussions

We derived the radial Schrdinger wave equation with Hulthen potential using Green's approach. To proceed with our approximation, we start with plotting the effective potential $V_{eff}(x)$ for different angular momentum quantum numbers l. As shown in Fig.1, minimum point of $V_{eff}(x)$ for $l = 0$ is a singular point. It exists only for $l \neq 0$ states and it increases considerably as the angular momentum increases. At this same time the well depth decreases as the angular momentum increases. Because the standard approximation is based on the expansion of the centrifugal term in a series around x_0, it is obvious that it could be valid only for the potential with a singularity point $x_0 \neq 0$,and accurate for values of x close to x_0, i.e., for low angular momentum energy states. The wave function corresponding to the ground state is shown in Fig.2, for different values of z. It is noted that for large values of deformation parameter, the wave function approaches towards the undeformed wave functions.

The difference between various methods become more apparent for large values of δ parameters and appear due to the approximation of the centrifugal term, which simply means that the better the accuracy in calculating energy eigenvalues the better the approximation of the centrifugal term, and hence the whole model.

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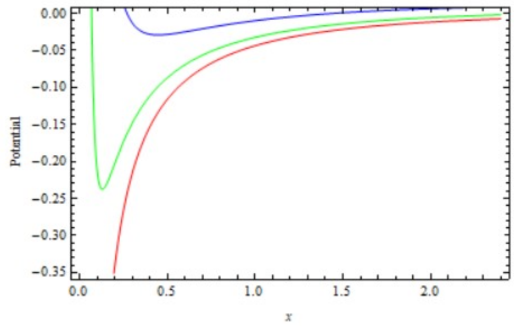

FIG. 1: Effective Hulthen Potential [Red→ l =0, Green→ l=1, Blue→ l=2]

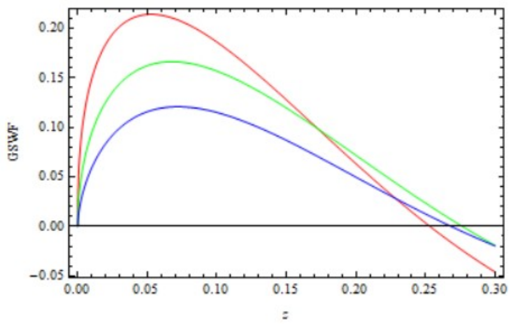

FIG. 2: The wave function corresponding to the ground state for different Values of z

References

[1] Ramazan Sever, Cevdet Tezcan, zlem Yeilta and Mahmut Bucurgat, International Journal of Theoretical Physics volume 47, pages22432248 (2008)
[2] D Agboola 2009 Phys. Scr. 80 065304
[3] A.N. Ikota, C.N. Isonguyob, Y.E. Chad-Umorena , and H. Hassanabadic,Acta Physica Polonica A,Vol. 127 (2015)