Confinement-Deconfinement transition and Z_2 symmetry in Z_2+Higgs theory

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Introduction

Z_N symmetry plays an important role in the confinement-deconfinement (CD) transition in pure $SU(N)$ gauge theories. In these theories, at finite temperature, the allowed gauge transformations are classified by the centre of the gauge group, i.e Z_N. Previous studies of Z_N symmetry in $SU(N)$+Higgs theories have found that the Z_N symmetry is restored in the Higgs symmetric phase in the continuum limit[1] (i.e. large number of temporal lattice points N_τ). In the Z_2+Higgs theory[2], the fields being Ising like, one can hope to understand better the Z_2 explicit breaking and it's dependence on the coupling between the gauge and Higgs fields and N_τ. Our goal in this study is to investigate the strength of the explicit breaking of this symmetry by varying the parameters of the theory and N_τ.

Z_2 symmetry in Z_2+Higgs gauge theory

The action for the Z_2+Higgs theory in four dimensional lattice ($N_s^3 \times N_\tau$) is given by,

$$S = -\beta_g \sum_P U_P - \kappa \sum_{n,\hat{\mu}} \Phi_{n+\hat{\mu}} U_{n,\hat{\mu}} \Phi_n. \quad (1)$$

where $U_P = U_{n,\hat{\mu}} U_{n+\hat{\mu},\hat{\nu}} U_{n+\hat{\nu},\hat{\mu}} U_{n,\hat{\nu}}$ is the plaquette with gauge links $U_{n,\hat{\mu}}$ and Higgs fields Φ_n take values ± 1. Here β_g is the gauge coupling and κ is the gauge Higgs interaction strength. The pure gauge part of the action is invariant under the Z_2 gauge transformations, $U_{n,\hat{\mu}} \to V_n U_{n,\hat{\mu}} V^{-1}_{n+\hat{\mu}}$, where $V_n = \pm 1 \in Z_2$. The V_n's satisfy the boundary condition, $V(\vec{n}, n_4 = 1) = zV(\vec{n}, n_4 = N_\tau)$ with $z = \pm 1 \in Z_2$. Under the Z_2 gauge transformation, Higgs field(Φ_n) in the fundamental representation transform as, $\Phi_n \to V_n \Phi_n$. Higgs fields are periodic and satisfy the boundary condition, $\Phi(\vec{n}, n_4 = 1) = \Phi(\vec{n}, n_4 = N_\tau)$. But the gauge transformed matter fields Φ_g satisfy the boundary condition, $\Phi_g(\vec{n}, n_4 = 1) = z\Phi_g(\vec{n}, n_4 = N_\tau)$, Φ_g does not remain periodic when $z = -1$. Therefore, in the presence of Higgs field Φ_n the Z_2 symmetry is broken explicitly.

Monte Carlo simulation results

The Polyakov loop $L(\vec{n}) = \prod_{n_4=1}^{N_\tau} U_{(\vec{n},n_4),\hat{4}}$ is the order parameter of this theory and transforms non-trivially under Z_2 gauge transformations as $L(\vec{n}) \to zL(\vec{n})$. Our simulation result shows that for pure gauge theory(κ=0) the order parameter shows a first order CD phase transition[3]. The CD transition is first order even in the presence of Φ(κ=0.13), though the transition point shifts to lower values of β_g. To check the N_τ dependence of the Z_2 symmetry at $\kappa = 0.13$, the distribution of Polyakov loop $H(L)$ is computed here in the deconfined phases(Fig. 1). For $N_\tau = 2$ the histograms clearly show there is no Z_2 symmetry but for $N_\tau = 8$, the histogram of Polyakov loop for two Z_2 sectors agree well with each other. The similar kind of result is obtained in confined phase as well. The simulation results indicate that the Z_2 symmetry is restored at large N_τ in the presence of matter fields.

The partition function and density of states in $0+1$ dimension

The temporal component of the gauge Higgs interaction corresponding to a particular spatial site can be written as, $S_{1D} =$

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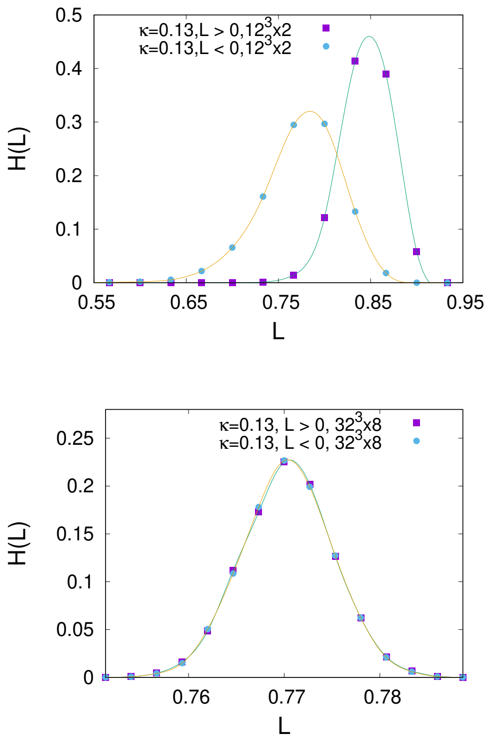

FIG. 1: Histogram of Polyakov loop L

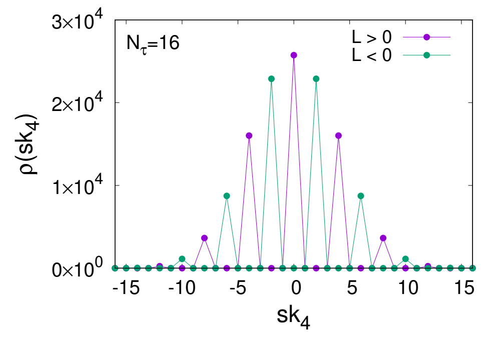

FIG. 2: *DoS* in 0+1 dimension

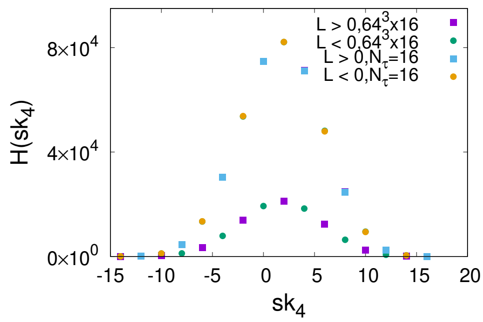

FIG. 3: $H(sk_4)$ fitted with 0+1D *DoS*

$-\kappa sk_4$, $sk_4 = \sum_{n=1}^{N_\tau} \Phi_n U_n \Phi_{n+1}$. The free energies corresponding to the partition function in the large N_τ limit in $0+1$D are given by, $V(L=1) = V(L=-1) = -TN_\tau \log(\lambda_1)$, where $\lambda_1 = e^{\kappa} + e^{-\kappa}$. This results show that there is Z_2 symmetry in $0+1$ dimensions in the limit of $N_\tau \to \infty$. The realisation of the Z_2 symmetry must come from the Z_2 symmetry of the entropy or the *DoS* i.e $\rho(sk_4)$. For large N_τ, $\rho(sk_4)$'s for both $L = \pm 1$ are well described by a gaussian(Fig. 2) centred at $sk_4 = 0$. The thermodynamics in the $N_\tau \to \infty$ limit will be dominated by peak height and distribution of $\rho(sk_4)$ around the peak, which is Z_2 symmetric, for all finite κ. Finally we try to fit the $3+1$ dimensional simulation result(histogram of sk_4 i.e $H(sk_4)$) for $\kappa = 0.1$ with $0+1$ dimensional DoS(i.e $\rho(sk_4)$) by including an extra Boltzmann factor, i.e $H(sk_4) \propto \exp(\kappa' sk_4)\rho(sk_4)$. The resulting fit(Fig. 3) agree very well with $H(sk_4)$. This result suggests that $3+1$D Monte Carlo simulations can be reproduced using the *DoS* of the $0+1$D model.

Acknowledgments

All of our numerical simulatons have been performed using the computational facility at IMSc, Chennai.

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